BLOOD BANK INVENTORIES

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INTRODUCTION

A blood bank is an institution for procuring, processing, storing and distributing blood.[1] Its primary objective is to ensure that the appropriate kind of blood is available when required for transfusion into hospital patients; it is thus an inventory facility. However, in view of the fact that blood has a limited shelf life—usually 21 days, but under certain circumstances 28 days—a second main objective is to minimize the amount of blood that expires, or becomes outdated, while held in the inventory. Clearly the bank will always have blood available if an infinite inventory is kept, and no blood will ever become outdated if a zero inventory is kept. To satisfy both goals simultaneously requires a compromise between these two extremes, and so it becomes a major problem to determine how this compromise can best be reached.

Apart from this central theme, the operation of a blood bank involves many other associated problems which, though each of great importance, nevertheless play a smaller role in determining how best to maintain a blood bank inventory. For each transfusion, the blood must be as appropriate as possible, i.e., of the right type and free of diseases such as serum hepatitis; it is preferable for it to be as fresh as possible when transfused. Until a decade ago the literature on blood banking was concerned solely with these associated problems. Over the last ten years, however, a literature on blood bank *inventory control* has come into existence. The questions that have been asked, and partially answered, are: (1) How can we best keep a record of all the operations going on in a blood bank? This is necessary to provide the proper blood as needed and to locate blood of rare types, as well as to evaluate the system as it is at present; (2) Suppose we change the system in various ways, what will happen? and (3) What is the optimal policy with regard to controlling the blood bank inventory?

BRIEF SURVEY OF THE LITERATURE

Since the literature on this topic is only some ten years old, and comprises less than 50 articles, an attempt at a comprehensive survey, though necessarily brief, is both feasible and worthwhile. This will be done by considering each of the above three questions separately. The more recent articles that are later reviewed in detail are not mentioned in this section.

Record Keeping

Some blood types are so rare and, thus, so little needed for transfusion, that it is not reasonable to stock them physically in a bank. For these bloods a "walking" blood bank is required, i.e., we must be able to locate donors of these rare types as the need for their blood arises. To do this some sort of central file must be kept: this may be done at a regional,[2] national[3-5] or international[6] level. While suggesting guidelines for the administration of an international panel, Mourant[6] had in mind beginning simply with a stenciled list, but foresaw the possibility of later turning to a mechanical or electronic system. The central file described by Greenwalt and Gajewski[3] uses punch cards for registration; those described as Guimbretière[2] and Moullec[5] are tied into an electronic computer system.

Even for the blood types that are not rare, it is helpful to have a file of all possible donors. The file should include information such as blood type, at what times the donor can or cannot come to donate blood, and when he last donated. This information enables a blood bank to supply relatively large quantities of fresh blood of a particular type for special purposes, e.g., open heart surgery, as well as to more easily replenish a low inventory for a particular blood type. In the former case the blood bank does not perform the function of an inventory, of course, but the function it performs is, nevertheless, an appropriate one for a blood bank. One system reported for filing such information[7] involves the use of plastic record cards, each donor card being punched according to the pertinent characteristics of the donor, and a mechanism using a light source that enables one to pick out all donors of a particular type. More recently various systems using regular IBM cards have been described,[8-11] and it has been suggested[12] that there is no reason why, in the not-too-distant future, all donor information on a national scale should not be stored in the memory of an electronic computer. Looking forward to the day when this will happen, Kempf[12] put forward a plea that all blood banks should use the same coding system for putting donor information on punch cards, or at least agree on just what information should be collected—a plea that seems to have gone largely unheeded so far. An interesting advantage that has been claimed for automating donor registers, using punch cards, is the ease with which personal letters can then be addressed to individual donors.[9,11] Not only can New Year cards be regularly sent, but each donor can be informed, in a letter either requesting a blood donation or thanking him for one, of the specific operation and patient for which his blood will be or has been used. The donor thus becomes more personally involved, and so is more willing to donate blood on subsequent occasions.

Whether or not donor files are kept, as soon as a unit of blood is drawn at a blood bank, some sort of bookkeeping is necessary to keep track of it until it is either transfused or outdated. Systems based on multi-part forms[13] or using a photocopying machine[14] have been described, but these cannot be used to perform statistical analysis in the way that automated systems can.[15-18] Allen[15] describes computer programs that allow frequent summarization of blood available in the bank, automatic billing techniques, and automated blood typing. A similar but more ambitious system[16] links a remotely located computer with a large blood bank and its member hospitals. Although the blood bank in this case is in no sense controlled by the computer,[19] the automatic record keeping makes it easy to summarize pertinent information daily or whenever required; and it is the easy availability of this information, properly utilized, that permits the bank to plan stock levels and inventory distribution more efficiently, thus reducing losses through outdating. A later article[17] analyzes the psychological, labor and economic problems involved in initiating such a program, and includes suggested guidelines for implementation. Recently, Stelloh[18] briefly described another prototype system, aimed at the same problem, in which flexibility and simplicity are stressed. Finally it should be mentioned that, if adequate records are available in a computer system, it is not a difficult matter to set up a program[8,20] to discover, with little probability of error, which donors are responsible for cases of transfusion hepatitis; Polissar[21] has shown how statistical decision theory can be used to solve this problem, and his method could very easily be adopted into any computerized blood bank system.

Blood Bank Models

If we wish to determine what would be the effect of changing a blood bank system in a particular way, without actually changing it, some sort of underlying model must be assumed. The earliest attempt to formulate a model for the operation of a blood bank was made by Sonnendecker;[22,23] this was soon followed by further models proposed by other members of the same research group.[24,25] Some of these models are quite unrealistic—for example, Millard[24] assumes that when blood is ordered from a central bank it arrives instantaneously—and in any case none of them seems to have been used extensively to determine the effect of changing the system.

Elston and Pickrel[26-28] studied this problem in a systematic way, using an electronic computer to simulate the operation of a hospital blood bank in which the total volume of blood involved is about 5,000 units annually. Their model will be described in some detail here, since it will be referred to again in the sequel.

Blood is assumed to enter the bank from one of two possible sources: (a) *random input*, or donor recruitment, which arrives in random amounts once a day and can remain in the bank 19 or 21 days before outdating; 19 days was considered as a possibility to allow for the time between the drawing of the blood and the result of the serology test, and also because of the time blood is held on reserve, or assigned, before being transfused; and (b) *orders*, which are placed with other banks and arrive one day later, and can remain in the bank 15 days. The blood that leaves the bank is called the *use;* a random use leaves the bank between each two consecutive time points, 24 hours apart, at which blood enters the bank. The amount of blood requested by physicians, which is on an average over twice as much as the use, is called the *demand*. The excess of demand over use in any one day results in blood being assigned and crossmatched, but not physically leaving the bank.

For each of the eight major blood types defined by the ABO and Rh systems, and for each day of the week, the random input, use and demand are each assumed to follow a negative binomial distribution; i.e., if y is the number of units of input, use or demand, we have

$$P(y) = \frac{(y+k-1)!}{y!\,(k-1)!}\, p^y(1-p)^k,\ y = 0,1,2,\ \ldots,$$

where P(y) is the probability of the number of units being y, and p and k are the parameters of the distribution: the values of p and k depend upon the blood type, the day of the week and whether y is an input, use or demand. The assumption of negative binomial distributions is justified both on empirical and, in the case of use and demand, on theoretical grounds.

Lastly, for the major results reported,[27] the model assumes that a very simple type of ordering policy is used: each day the number of units of blood in the bank of a given type is determined, and further units are ordered, if necessary, to make this up to a prespecified level S. Thus if x is the number of units in the bank (whether or not any of the units are assigned), the amount ordered is $\max(0, S - x)$. The particular value of S chosen depends upon the blood type, since the volume of blood used depends upon this. For the general results given, S was taken to be that level that minimized the following *loss function:* the sum of 25 times the number of units by which use would be expected to exceed what is available in the bank and four times the number of units expected to outdate. On this basis the study determined what would happen if, (when the oldest blood in the bank is always used first)

1. the random input is unchanged,
2. the random input is doubled, or
3. the random input is eliminated entirely.

Furthermore, for 1, the effect of always using first the freshest blood in the bank was studied. In all cases the values of the parameters p and k were taken as those applicable to one particular hospital blood bank, but there is no reason to doubt the general applicability of the qualitative aspects of the results. One characteristic of the blood bank simulated that should be particularly noted, however, is that, when the random input is unchanged, it is about one and a half times as large as the ordered input.

The results of the study may be briefly summarized as follows. When the system is unchanged, the ordering policy used (and in particular the choice of S) leads to acceptable losses; i.e., there is an acceptable balance between the number of units outdating and the number of units that the bank cannot supply when needed. If the random input is doubled, the losses are almost quadrupled. The losses would be least if the input were completely ordered and the life of the ordered blood were not restricted; but when the ordered blood has a life of only 15 days the losses are somewhat higher than when the random input remains unchanged. The use of the freshest blood first will, of course, lower the average age of the blood transfused, but it will lead to an excessive amount of blood outdating; we may expect to gain from such a policy only if we are in a situation where we are in any case forced to lose a large percentage of blood by outdating.

Very recently, in two preliminary papers,[29,30] a more analytical approach to modeling some blood bank operations has been put forward. This uses the technique of absorbing Markov chains and shows, with more mathematical rigor, the advantages of certain changes in blood bank operating policies. In particular the policy of assigning the oldest blood to one particular patient to meet a demand, regardless of the probability that it will actually be used, is criticized. It is proposed that the age and number of units assigned in each case should depend upon the probability of the blood actually being used for transfusion and that, in some situations, when the probability of transfusion is low, the same unit should be assigned simultaneously to more than one patient. Such changes in the system will clearly reduce the effect of demand being greater than the amount of blood available in the bank, and to some extent will also lessen the amount by which use is in excess of what is available. Another mathematical model has been proposed to study the effect of changing the issuing policy:[31] it can be used to study a broad class of such policies, including as special cases the use of the freshest blood first and the use of the oldest blood first. In particular, formulae are developed to determine, subject to certain restrictions, how many units of each age level should be issued to meet each demand; it is too early yet to say how useful these methods may be in the actual operation of a blood bank.

Optimal Ordering Policies

Whereas the major part of the literature surveyed so far has the primary objective of helping the director of a blood bank decide how much blood he should keep on hand in his bank, either by giving him summary statistics about his bank or by telling him what will happen if he keeps various levels on hand, there has been little written directly on the problem of what is the optimal ordering policy. Some early solutions to this problem[32,33] are probably adequate when a large volume of blood is involved, but are of little use to small hospital blood banks, or even to larger banks for the rarer of the major blood types; in practice, the problem becomes acute for a blood type involving less than 1,000 units annually. Thus, one of these solutions[33] essentially uses a deterministic model of a blood bank and, hence, describes a simple rule-of-thumb method to find the correct order to place with a central bank. It is pointed out that the optimal ordering policy must depend upon the age distribution of units already in the inventory, not simply upon the total number of units there. However, this had been previously noted in a paper[34] that showed, in one particular case at least, that knowledge of the age distribution of units available does not appear to be materially useful in determining the order to place. This result is not too surprising when one considers it was found using a model in which the oldest blood is always used first and in which orders are placed daily, arriving one day later. In such a situation the optimal ordering policy must depend largely upon (and perhaps solely upon) two quantities only: the total amount of blood in the inventory and how much of it will become outdated, if not used, within the next 24 hours.

A simple and approximate method of determining an ordering policy suitable for hospital blood banks was first proposed by Silver and Silver.[35] Inventory control levels are determined by establishing a range of minimum and maximum units for each blood type, based on two simple calculations: (1) the use and outdating over semimonthly periods; and (2) the occurrence of large emergency demands over two consecutive days during the semimonthly periods.

Later, Elston and Pickrel[36] noted that the model they used in their previous paper[27] gives results that are, for all practical purposes, the same as those obtained if the mean use and input are considered to be independent of the day of the week. This makes it much simpler to construct general purpose tables showing the results of the simulation when the bank is kept at different levels, since it is no longer necessary to take into account the many possible ways in which the same weekly use of input can be distributed among the seven days of the week. Therefore, using the same assumptions as have already been stated above, they tabulate the following three characteristics, in terms of yearly means, for a variety of situations:

A. the number of days on which excess use occurs

B. the number of units not supplied, and

C. the number of units becoming outdated.

The situations they consider are: mean daily random input 0, 1 or 2 units; mean daily use between 0.6 and 3.0 units; and inventory level S, between 2 and 26 units. They show how their tables can be used to calculate the optimal inventory level using any given loss function that can be specified in terms of A, B and C. (Their earlier paper assumed the particular loss function 25B + 4C, as explained above). It should be noted that the results depend upon estimates of p obtained from one particular blood bank, and, even though the means of the distributions used are undoubtedly much more important in determining the results than the particular value of p used, it would be nice to know that the parameter p does not change much from bank to bank. Unfortunately, it does not seem to have been estimated for any other bank; indeed, although distributions of blood input, use, and demand have now been studied in several places, there are no other reports of anyone trying to fit negative binomial distributions to them.

In view of the fact that the use of citrate-phosphate-dextrose solution allows the safe storage period of human blood to be prolonged from 21 days to 28 days,[37] Elston[38] later calculated an analogous set of tables appropriate for 28-day shelf life, including a table of optimal inventory levels (and the resulting characteristics) for the loss function 25B + 4C.

SELECTED RECENT ARTICLES

In this section six recent articles will be reviewed. One of them (Jennings, 1968) summarizes some results of a simulation study that is described in more detail in an earlier report.[39] For the purposes of this review certain details that are not explicitly stated in the article, but which can be found in the original report, will be discussed as though they are given in the article itself. As in the survey above, it will be convenient to consider the six articles under three subheadings, two articles under each, though it should be clear that some of the articles could really be considered under more than one subheading. The subheadings are somewhat different from those used in the previous section but, nevertheless, reflect an attempt to classify each article on the basis of its major emphasis—or: record keeping, modeling or ordering policy.

The Computer in the Blood Bank

The two articles that will now be reviewed deal with the same problem: that of using a computer to keep records and summarize information for a hospital blood bank; in this respect, they differ fundamentally from an earlier report.[17] which describes how a computer is used to maintain records for a blood collection and distribution center. Both reproduce within the computer counterparts to the numerous ledgers that are otherwise maintained manually, without needing any increase in personnel; in fact, in one case we are told "about half of the time previously devoted by our secretary to record keeping is now available for reassignment." The goals of both computer programs are similar, and are summarized in Table 1. The volume of blood involved is about the same in each case: in one case, that of the University of Kentucky Medical Center blood bank, blood is provided for about 6,000 to 7,000 transfusions per year; in the other case, that of the blood bank at the Yale—New Haven Hospital, there is an annual use of about 8,000 units of whole blood, 3,500 units of packed red blood cells, 4,200 units of plasma and cryoprecipitate, and 3,500 units of platelets. But the details of the two programs are different in many respects.

Stewart and Stewart (1969), at the University of Kentucky, describe a computer program written in Fortran IV that could be used on any computer equipped with random disk file access capability. It is currently used on an IBM 1800 Data Acquisition and Control System (2 micro-second access time) with a 1443 printer, 1442 card reader, and 1816 keyboard typewriter; there are two 2401 magnetic tape drives and three 2310 disk units. It takes about 30 minutes a day to prepare the punch cards to enter the data into the computer, and the program uses about six minutes of machine time daily. An additional 10 to 15 minutes are used about once every ten days to print completed files and transfer them to magnetic tape.

Five kinds of information can be entered into the computer, and a different type of data card is used for each: the information may be on donor units, intrabank transfers, transfusions, receipts and audit control. The donor unit cards contain: unit identification number, donor's name, source of the blood, blood group, date of expiration, if rejected for any reason, hospital number of the patient to receive credit for replacement donation and anticoagulant. From these records a file of usable blood is obtained and printed. While in the bank a unit of blood may undergo any of 22 intrabank transfers, as listed in Table 2. Cards

TABLE 1

Goals of Computer Program*

Provide better records
- Blood and blood product used
- Blood outdated
- Blood discarded or transferred

Provide an inventory listing
- Up-to-date and accurate
- Easily produced
- Easy to use

List blood that is approaching expiration date
- Alert staff to possible outdating

Reduce outdating

Reduce shortages by improved ordering

Add *no* additional personnel

Be compatible with remaining hand records

* From Bove, J.R., and McKay, D.K., *Transfusion*, 9, 143, 1969. With permission.

TABLE 2

Intrabank Transfers*

1. Borrowed donor blood
2. Loaned donor blood
3. Units sent from other hospitals
4. Units collected for other hospitals
5. Outdated units
6. Quarantined units
7. Quarantined and reissued
8. Quarantined and discarded
9. Discarded for any reason
10. Heparinized blood transferred to ACD
11. Reserved for a patient
12. Taken from patient reserve
13. Returned to sender
14. Preparation platelet rich plasma
15. Preparation platelet concentrate
16. Preparation fresh frozen plasma
17. Preparation cryoprecipitate
18. Preparation stored plasma
19. Preparation salvaged plasma
20. Transfer quarantined stored plasma to use
21. Preparation of bank plasma (for Factor IX)
22. Split unit into partial units

* From Stewart, R.A. and Stewart, W.B., *Transfusion*, 9, 78, 1969. With permission.

indicating such transfers include the donor unit number and the transfer code. Transfusion cards contain: hospital number and name of recipient, location, medical service, blood types of recipient and donor, unit number and product code (12 different products may be released for transfusion). A file of transfused units is prepared and printed; and eventually, after correction if necessary, a completed donor file is prepared that combines both donor and recipient information: this is both printed and stored on magnetic tape. Receipt cards contain the code of the institution from which blood is received, the hospital number of the patient to be credited and the amount of the transaction. These data are printed daily to help in the ordering of blood from regional centers. The audit control cards contain the donor numbers and blood types of all units present in the bank; these can be presented daily or at less frequent intervals and, using them, the computer checks for discrepancies with all other information that has been submitted. Over a hundred different errors can be detected in this way, and any error that is so found can be corrected by supplemental independent programs.

Given all this information, the computer prepares inventory and statistical reports. A daily report lists all units that have been in the bank during the last 24 hours, categorized as indicated in Table 3. Within each class the units are grouped by blood type and listed in the order of their outdate. The following is also printed for each blood type: the units within two days of outdating; the amount of blood available and average number of days to outdating; and a chart comparing the number of units used and received and previous average daily use. A weekly report gives a list of "on call" donors who have not donated in the last eight weeks. Further reports can be prepared monthly (or as desired).

In their discussion of this computer system, the authors note its several advantages. About four significant errors per week are discovered in the recording of blood transfers, most of which would probably have passed unnoticed before. The daily statistical summary is an aid to anticipating blood shortages, though it does not seem to have significantly affected the amount of loss due to outdating. The monthly statistics are now available on the first of the month, instead of being two or three weeks late. It is thus clear that their program has successfully answered the bookkeeping problem at a cost that they estimate to be about $400 per month.

TABLE 3

Inventory Classification of Whole Blood*

1. Whole blood transfused
2. Packed cells transfused
3. Split unit transfused
4. Outdated unit
5. Discarded unit
6. Unit collected for other hospital
7. Loaned to other hospital
8. Disposed for any reason
9. Whole blood available for use
10. Packed cells available for use
11. Split unit, both parts available
12. Split unit, one part available
13. Unit outdated, but not deleted
14. Reserved or quarantined
15. Other classification

* From Stewart, R.A. and Stewart, W.B., *Transfusion*, 9, 78, 1969. With permission.

At the Yale—New Haven hospital, Bove and McKay (1969) tackled the same problem, but their methods differ in several important respects. They decided to use a relatively small laboratory computer (IBM 1130), rather than to share time on a larger computer available to them. This is usually less advisable in that it often leads to greater costs; but, of course, local conditions often dictate what the "best" choice is. Their system includes a typewriter-printer and keyboard, and one disk is used to store the blood bank programs and data files; as in the other system just described there is a line printer and a 1442 card reader. The program uses about 10 minutes of computer time daily.

Only one kind of punch card is used for entering all data into the computer. This card, one for each unit of blood, is furthermore used for three other purposes: 1) as a working inventory in the blood bank; 2) as a sign-out receipt for blood leaving the bank; and 3) as a legal record of the disposition of each unit of blood. However, it is convenient to produce a duplicate card for each unit of blood: one card is then used for entering data into the computer and as a working inventory; the other stays with the unit of blood.

The following information is punched into the card: blood type of the donor unit, unit number, the component or fraction and date of expiry; also, if desired, a code may be used indicating the source if it is received from another hospital. Cards with this information are run through the computer daily and are then placed in a special file that allows technologists to easily read the number, blood type, expiration date and other information. When blood is assigned on crossmatch the appropriate card is moved from the "off reserve" file to the "on reserve" file. (One section of the card is used as a *release record,* indicating when the blood was signed out, by whom and for which patient.) Information on blood usage is hand-punched, (using a Port-A-Punch), at a later stage, to indicate what finally happens to each unit, including, for example, the clinical service that uses it.

Given this information on each unit, several summaries are printed. One indicates the number of units of each component type (whole blood, platelets, etc.) used by each clinical division; another classifies the units by component type and blood type. An inventory printout lists each unit by blood type; the whole or selected portions only can be printed as desired. A list of units approaching outdating classifies such units as 1) outdate today; 2) outdate in two days; or 3) outdate in five days. Weekly, monthly and yearly summaries can be obtained. Error messages occur when unacceptable information is entered, and a unit that outdates in the machine is listed as *unaccounted for* if its final destiny is not appropriately entered into the machine. Because of this, *unaccounted for* units suddenly became common when this program was used, indicating that the previous record keeping had made it easy to issue items without properly recording the fact.

One particular report given by this computer program, and not by the one at the University of Kentucky, is a calculated blood order for the day. For each blood type the actual inventory is subtracted from a previously determined optimal level; several approaches are used to determine this level, but, unfortunately, no details of these methods are given in the article.

The main feature of this system is the "one card—one unit" concept, as opposed to the five different kinds of data cards used in the first system described. It is impossible to pass judgment as to which system is better, since so much depends upon local conditions. Both programs have proved their merit for the purpose of keeping accurate records, but it should be carefully noted that the second program has built into it an attempt to have the computer make a major decision, namely, how much blood to order. It is only when the computer is used to help directly in this kind of decision making that full use will be made of its capabilities. We shall return later to discuss this aspect of computing in the blood bank.

Comparison of Policies

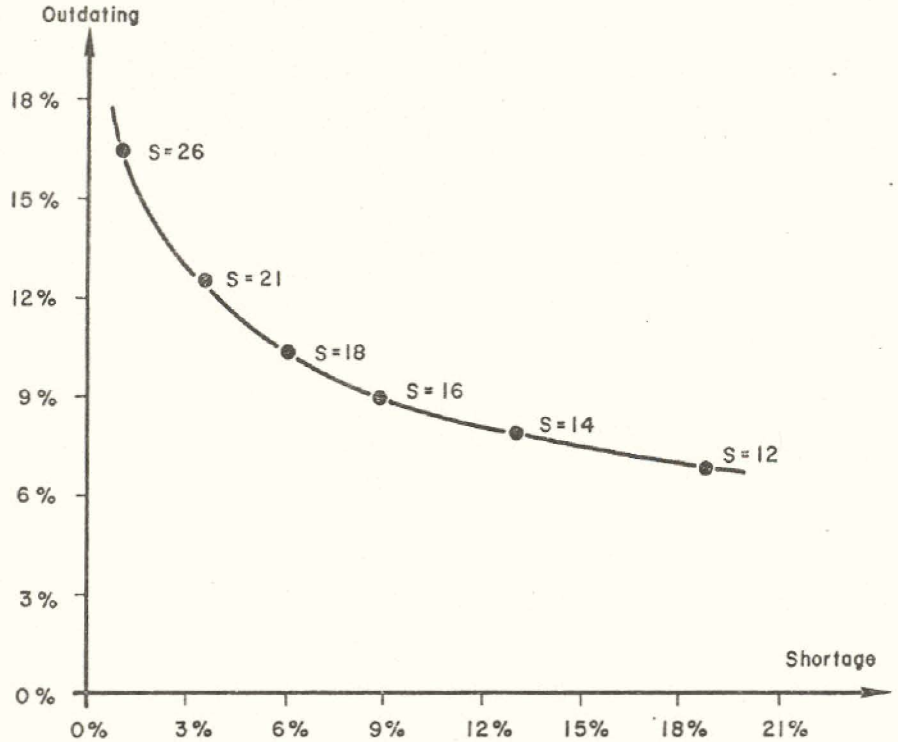
The paper by Jennings (1968), which will now be discussed, has goals and methods similar to the earlier paper by Elston and Pickrel[27] that has already been considered. One basic difference, however, involves the distinction between assigned and unassigned inventories. Whereas Elston and Pickrel recognized the distinction between use and demand, they simulated a "one-compartment" model of blood bank operations. Jennings, on the other hand, simulates a "two-compartment" model, the two compartments being *assigned* and *unassigned* inventories, respectively. This is, of course, certainly nearer to reality. In fact, in many other respects also his model is nearer to reality: he uses empirically found distributions rather than theoretically fitted ones. The result is that he simulates in great detail just one blood type for one particular hospital; and it is difficult to see how his model can ever be modified, in the way Elston and Pickrel[36] later modified theirs, to be of general applicability in other blood banks. It is a sad fact that, at the present time, we are faced with the dilemma of choosing between a more realistic model of very narrow applicability and a less realistic model of very wide applicability.

Apart from this basic difference in the model, Jennings also considers a distinctly different goal for a blood bank and, hence, also a distinctly different ordering policy. He implicitly takes the primary objective of a blood bank to be to ensure that the appropriate kind of blood is available for assignment when demanded, not that it should simply be available when required for use. In the latter case it is clear that the bank only "fails" when use exceeds what is available; Jennings considers the bank to have failed as soon as demand exceeds what is available in the unassigned part of the inventory and, in fact, defines *shortage* as this excess of demand over unassigned inventory. For this reason he considers a method of placing orders that depends only upon the total number of units in the unassigned inventory. Similarly, whenever demand exceeds the amount in the unassigned inventory, this excess is assumed to arrive immediately as a special *shortage order* from some other source, in the form of fresh blood, and this is then placed in the assigned inventory. Elston and Pickrel assume a similar special input to the bank only when *use* is in excess of the total inventory (and since this occurs only rarely they obtain virtually the same results whether this particular model, or a model in which the bank can contain a negative amount of blood, is simulated—this latter representing what happens when there is postponement of elective surgery).

In the study under consideration a relatively large hospital blood bank is simulated, the Peter Bent Brigham Hospital blood bank, and the blood type studied (B,Rh positive) accounts for about 1,000 units transfused annually. Empirical probability distributions are used for: the percentage of the daily order, and its age, received from the central blood bank (the rest of the order comprising fresh donor blood); the demand, use, and release of assigned blood back to the unassigned inventory; the length of time spent on reserve by each unit eventually released (depending on the age of the unit); and the random input. In the basic model orders are placed once a day, using the following ordering policy: if x' is the total number of units in the unassigned inventory, the order placed is $\max(0, S - x')$, where S is prespecified, as before. However, this order is considered as being made up of two parts: one part arrives immediately, being blood from the central bank that is on an average 5.5 days old when it arrives; the other arrives as fresh donor blood just after half of the day's total demand has been put into the assigned inventory. Since, overall, 59% of the ordered blood comes from the central bank, on an average the ordered blood has a shelf life of $17\frac{3}{4}$ $(= 0.59(21 - 5.5) + 0.41(21) = 9.145 + 8.61 = 17.755)$ days on arrival; furthermore, it arrives on an average with a much smaller delay than that (one day) assumed by Elston and Pickrel. The results, for varying values of S, are then plotted against outdating and shortage as illustrated in Figure 1. In actual practice, for the bank simulated, the value of S lies between about 15 and 18 units, and a comparison between the results of the simulated model at these values of S and various statistics for the actual blood bank suggests that the correspondence between the two is reasonably good.

Apart from the effect of changing the inventory level, as shown in Figure 1, the effects of the following further policy changes in the model are studied: variation in the inventory level; reduction of the variability in the age and amount of blood received from the central blood bank; increase in the shelf life of incoming blood; maintaining a minimum inventory level or emergency reorder point; and placing orders twice a day. The basic results of the study will now be listed.

If the inventory level S fluctuates from day to day, then for frequent but small fluctuations the result is virtually the same as when a constant average value is maintained: e.g., alternating between levels of 18 and 16 units daily leads to approximately the same results as keeping the level constant at 17 units. When the variations in S are large and frequent (e.g., daily alternation between 12 and 26 units), the result is virtually the same as when a constant value somewhat greater than the average is maintained (21 units, rather than the average value of 19 units, for the example given). When the variations are infrequent (e.g., monthly), the shortage and outdating can be found by averaging the individual values of shortage and outdating. Thus, frequent fluctuations lead to points on the curve of Figure



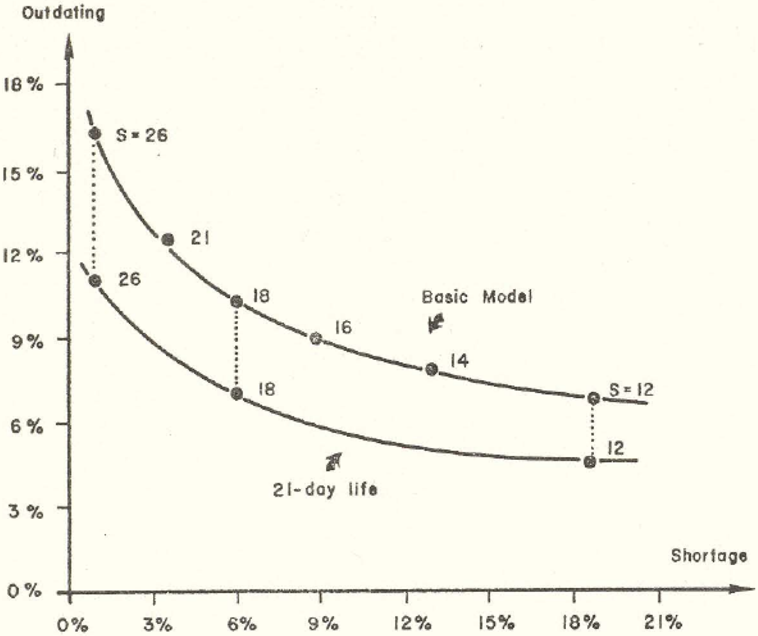
Outdating and shortage for the basic model when the inventory level S varies between 12 and 26 units. Adapted from Jennings, J.B., *Transfusion,* 8, 335, 1969. With permission.

1 between the extreme values of S, while infrequent fluctuations lead to a point on a straight line connecting two points on this curve.

The percentage of ordered blood that is fresh donor input, as opposed to coming from a central bank, is a random quantity in the basic model. Variability in this quantity is studied by determining what happens when all donor input is the same fixed average value of 41% of the total order every day. Similarly, variability in the age of ordered blood is studied by determining what happens when all blood ordered from the central bank has the same fixed average age of 5.5 days when it arrives—this is almost the same, for 21-day shelf life, as the 15-day life assumed for ordered blood by Elston and Pickrel. In each case it is found that eliminating the variability leads to a very small reduction in outdating.

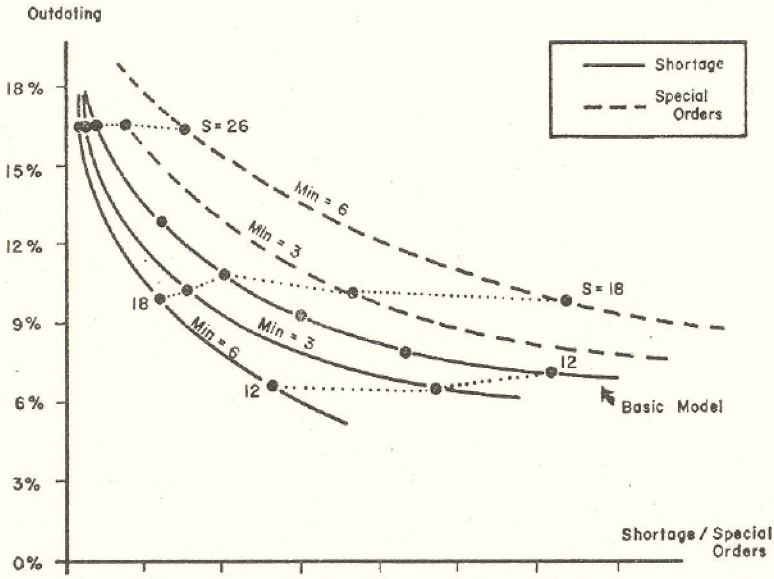
Increasing the life of ordered blood naturally leads to fewer losses; i.e., the curve shown in Figure 1 is displaced downwards as indicated in Figure 2. The particular comparison here is between the basic model and one in which all ordered blood has a shelf life of 21 days on arrival.

The effect of maintaining a minimum inventory level is studied by considering two kinds of special orders, which always result in fresh blood being delivered instantly. The special orders are to be distinguished from the daily orders that make up the ordering policy, by which blood is received from a central bank. One kind of special order is the *shortage order* which has been defined for the basic model and occurs as soon as demand is in excess of unassigned inventory; the second kind is a *safety order,* non-existent in the basic model, which occurs as soon as the unassigned inventory is below a certain minimum level. Thus, the new model differs from the basic model in that whenever the inventory goes below the specified minimum level the order that brings the bank up to that level is delivered instantly and consists of fresh blood; the regular daily order up to the level S, as before, consists of two parts: part follows an empirical age distribution and comes immediately; part is fresh but comes later. The effect of maintaining



Outdating and shortage for the basic model and for the case of all ordered blood having 21 days of shelf life. Adapted from Jennings, J.B., *Transfusion*, 8, 335, 1969. With permission.

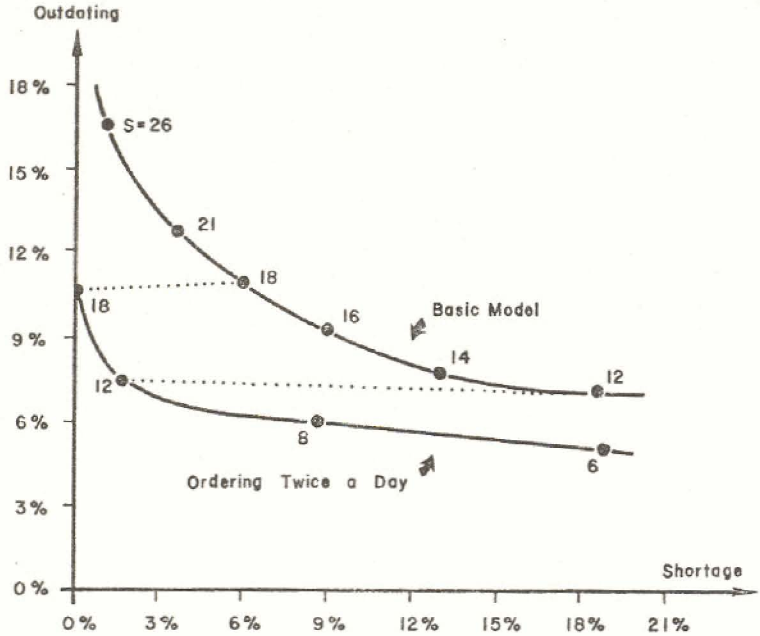
FIGURE 3



Outdating, shortage and special orders for the basic model and for the cases of three- and six-unit minimum inventory levels. Adapted from Jennings, J.B., *Transfusion*, 8, 335, 1969. With permission.

minimum inventory levels of three and six units is shown in Figure 3. The solid lines are outdating plotted against shortage, as before; the dashed lines indicate the increase in special orders brought about by the introduction of safety orders (in the basic model the solid and dashed lines coincide, since the only special orders are the shortage orders). Figure 3 shows

FIGURE 4



Outdating and shortage for the basic model and for the case of ordering twice a day on weekdays. Adapted from Jennings, J.B., *Transfusion*, 8, 335, 1969. With permission.

the trade-off between reduced shortage on the one hand and the increased special orders on the other hand, resulting in extra cost and inconvenience. It should be noted also that, for fixed S, changes in the minimum inventory level have little effect on the percentage of outdating.

The effect of placing orders twice on every weekday, morning and afternoon, is shown in Figure 4. We can view the basic model as one in which the afternoon order is 0, so the figure indicates that as the afternoon order rises from 0 to $\max(0, S - x')$ the curve is shifted to the left. This is, in fact, confirmed when a model is studied in which the morning order is $\max(0, S - x')$ and the afternoon order is $\max(0, S' - x')$, in which S' lies between 0 and S. Thus, provided the second ordering level S' is no greater than S it has virtually no effect on the percentage of outdating. We, therefore, conclude that it is the maximum inventory level, rather than any minimum or average value of it, that largely determines the amount of outdating.

All these various results, obtained by simulating in detail the operations of a particular blood bank, are of great value in indicating the general qualitative nature of the changes to be expected on controlling or altering various parameters of any blood bank operation. The quantitative nature of the changes, however, is applicable to the particular hospital studied alone. For this reason it is impossible to compare the numerical results with those obtained by other workers. Even the study by Elston and Pickrel, which in many aspects is a similar simulation study, is very different in the details assumed for the operation of the bank. We, therefore, turn now to the other extreme in modeling, a general mathematical formulation for which it is relatively simple to obtain analytical results. We shall see that this has both advantages and disadvantages over a detailed simulation study.

Pegels and Jelmert (1969) consider a stationary Markov chain model for a blood bank. Each unit of blood is considered, at any point in time, to be in one of 23 different states. State I is "transfused"; state II is "expired" or "outdated." The other 21 states, labeled states 0,1,2, . . . ,20, represent the different ages of blood in the bank; thus blood that is not yet one day old is in state 0; blood that is one day old is in state 1; blood that is two days old is in state 2, etc. The essential feature of a stationary Markov chain, which is quite realistic in this situation, is that the probability that a unit of blood should move from any one state to any other does not depend upon the particular time point (day) considered. Thus, the whole system is completely described by just a single set of 23×23 probabilities p_{ij}, p_{ij} being the probability that a unit of blood should move to state j at each time point, given that it is in state i. States I and II are called *absorbing* states, in that once a unit of blood is in one of these states the probability is unity that it should stay there. Most of the p_{ij} are zero, since blood that is i days old (i.e., in state i) can go into one of only three states at each time point—state I, state II or state $i+1$; and, in fact, state II can only be reached from state 20 (or, since once there it always stays there, from itself). Thus, the only p_{ij} that are not zero are: $p_{I\,I} = 1$, $p_{II\,II} = 1$; $p_{i\,I}$ and $p_{i\,i+1}$ for $i = 0,1,2, \ldots, 19$; $p_{20\,I}$ and $p_{20\,II}$. Furthermore, since $p_{i\,I} + p_{i\,i+1} = 1$ for $i = 0,1, \ldots, 19$ and $p_{20\,I} + p_{20\,II} = 1$, the Markov chain that represents a blood bank system is completely specified once the 21 "transfusion probabilities" $p_{i\,I}$ ($i = 0,1,2, \ldots, 20$) are given.

To proceed further with this model, a knowledge of matrix algebra and elementary probability theory is necessary. Let **Q** be the 21×21 matrix (p_{ij}), $i,j = 0,1,2, \ldots, 20$, and let **S** be the 21×2 matrix (p_{ij}), $i = 0,1,2, \ldots, 20$, $j = I, II$. Furthermore, let $\mathbf{T} = (\mathbf{I} - \mathbf{Q})^{-1}$, where **I** is the 21×21 identity matrix. It can then be proved that **TS** gives the probabilities of a unit in a given state ultimately ending up in the absorbing states I or II, i.e., the probabilities of either being transfused or being outdated. Another useful result that can be proved is that **Te**, where **e** is a 21×1 column vector of ones, gives the expected number of days it will take for blood in a given state (i.e., of a given age) to end up in an absorbing state. If we add i to the i-th element of this vector ($i = 0,1,2, \ldots, 20$), the result is the expected age of each unit of blood upon arrival in the absorbing state, given that it is i days old to start with.

Now for $i = 0,1,2, \ldots, 20$ and $j = I, II$, let x_{ij} be the expected number of days required for a unit in state i to end up in state j; and let r_{ij} be an element of **TS**, the probability that a unit in state i should end up in state j. Then the i-th element of **Te**, the expected number of days required for a unit in state i to end up in an absorbing state, is clearly equal to

$$r_{i\,I}x_{i\,I} + r_{i\,II}x_{i\,II}, \qquad i = 0,1, \ldots, 20.$$

Thus, if we calculate **Te** and **TS** we can set up twenty-one equations in which the x_{ij} are the unknowns; and since we know $x_{i\,II} = 21 - i$ (a unit in state i requires exactly $21 - i$ days to end up in state II, outdating), these 20 equations can be solved for the 20 quantities x_{iI}; and $x_{iI} + i$ is the expected age of a unit in stage i when it eventually reaches the transfusion state. Thus, if we wish to know, for a unit of fresh blood entering the system ($i = 0$), the probability that it should be transfused and its expected age at transfusion, the appropriate values are r_{0I} and x_{0I}, respectively.

Two simple artificial examples illustrate how this model can be used. In the first example, which represents a tendency to use the oldest blood first for transfusion, we let the vector of transfusion probabilities be given by $p_{iI} = 0.01 + i/100$. In the second example, which represents a tendency to use the freshest blood first, the vector is reversed, i.e., we let $p_{iI} = 0.21 - i/100$. These vectors are shown in Table 4 (see next page).

Using the mathematical procedure described, we find that in both cases the probability of transfusion (r_{0I}) is 0.9176; so the probability of outdating is in both cases the same, $1 - 0.9176 = 0.0824$, or about 8%. However, for fresh blood entering the system, the expected age of blood when transfused (x_{0I}) is six days older when there is a tendency to use the oldest blood first—11.9 versus 5.9 days old. It must, of course, be understood this gain is not the whole story. Elston and Pickrel[27] showed that using the freshest blood would necessarily increase the amount of outdating in a system in which the excess use has to be kept nearly constant. In the system modeled here the amount of outdating has been kept constant, and the result is that a tendency to use the freshest blood first will automatically increase the amount of excess use. This can best be seen

TABLE 4

Probability of Transfusion, p_{iT}, of Blood i Days Old.

i	Example 1	Example 2
0	0.01	0.21
1	0.02	0.20
2	0.03	0.19
3	0.04	0.18
4	0.05	0.17
5	0.06	0.16
6	0.07	0.15
7	0.08	0.14
8	0.09	0.13
9	0.10	0.12
10	0.11	0.11
11	0.12	0.10
12	0.13	0.09
13	0.14	0.08
14	0.15	0.07
15	0.16	0.06
16	0.17	0.05
17	0.18	0.04
18	0.19	0.03
19	0.20	0.02
20	0.21	0 01

by studying the average inventory level for the two examples.

The probability that blood in state i should not be transfused is $1 - p_{iT}$. Thus, if the average inventory level of blood in state i is I_i, we can write

$$I_i = (1 - p_{iT})\, I_{i-1},\ i = 1,2, \ldots ,20$$

and I_0 is $(1 - p_{0T})$ times the average daily inflow of fresh blood into the system. Thus, if we let the average daily inflow of fresh blood be I, the average inventory totaled over bloods of all ages is simply

$$\sum_{i=0}^{20} I_i = I[(1 - p_{0T}) + (1 - p_{0T})(1 - p_{1T}) + (1 - p_{0T})(1 - p_{1T})(1 - p_{2T}) + \ldots + \prod_{i=0}^{20} (1 - p_{iT})]$$

Arbitrarily setting I = 100, the total average inventories for the two examples turn out to be 1098.1 units when the oldest blood is used first and 516.5 units when the freshest blood is used first: it is thus clear that the latter system must lead to much more excess use over what is available in the system.

In the same paper Pegels and Jelmert briefly indicate how the same kind of model can be used if we wish to simulate a two-compartment system. For each age, blood can be in either of two states, assigned or unassigned; and provided all the transition probabilities, i.e., the probabilities that blood should move from any one state to any other, are specified, exactly the same calculations can be made. To simplify the problem they consider in this case two-day periods instead of one-day periods, and give a further artificial example. Clearly, more transition probabilities have to be given to completely specify the system, but the arithmetic procedure is exactly the same.

The general mathematical formulation that has just been described is obviously of wide applicability and use if we wish to compare the results of different systems. Given the inflow of fresh blood and the 21 transfusion probabilities we can calculate the average inventory, the expiration rate and the average age of blood at transfusion. The method can easily be extended to allow for the inflow of blood of various ages, and to allow for a two-compartment model. It has, however, the grave disadvantage of telling us nothing precise about the probability of use (or demand) being in excess of what the system can supply. This is clearly a function of the average inflow, which determines the inventory level. So if we know what inflow, or inventory level, gives an acceptable probability of being short of blood, this can be used to determine the other characteristics of the system. All this assumes, of course, that the 21 transfusion probabilities are known. These will be different for each blood bank, and it is by allowing for these as the input parameters that the model has such general applicability. To a certain extent, the transfusion probabilities are determined by the daily distribution of blood use, but their relative sizes are flexible, depending largely on whether it is desired to issue fresher or older blood first for transfusion. In a recent preliminary paper, Pegels[31] presents a mathematical approach to the appropriate choice of these probabilities, given the desired average inventory level.

Determining What Order to Place

As indicated earlier, little has been written directly on the problem of determining how much blood should be requested in the daily orders. We shall now consider two recent papers by Pegels (1969a, 1969b) in this area. There is a certain amount of overlap between the two papers, so both will be considered together without any attempt at identifying the papers individually. These two papers appear to be the only recent papers directly concerned with the problem of how much to order, and they are analytical in nature; unfortunately, the assumptions made are quite unrealistic for small hospital blood banks, but it is these very assumptions that make an analytic approach reasonably simple. The solution is, in fact, basically the same, with extensions, as that put forward by Rockwell et al.,[32] in their case specifically to meet demand on the unassigned inventory. Pegels does not distinguish between use and demand, and uses the term "demand" more in the sense that we have kept here for the word "use." Although, with suitable modification, his general approach to ordering blood could be used to satisfy either demand or use, he does not consider the details of a two-compartment model; we shall, therefore, here substitute the word "use" wherever he uses the word "demand."

First, consider the case where there is no random input. The basic problem is considered as that of determining at what inventory level S should be maintained for a particular blood type given a specified maximum probability that use should be in excess of what is available. Denote this probability by γ; then S is chosen such that γ is the probability that use should be greater than S. It should be noted that if this level S is used, as before, to determine an order size $\max(0, S - x)$, then, for this choice of S to satisfy the specified probability that use should be in excess, it is implicitly assumed that all orders arrive immediately with no time lag. This important point is not stated explicitly in the articles being reviewed, but it is a major simplification, the effect of which should not be underestimated. To find S we need to know, of course, the distribution of the use for the particular blood type, this distribution being assumed not to depend upon the day of the week. If use is normally distributed with mean μ and standard deviation σ, then we have quite simply

$$S = \mu + z_\gamma \sigma$$

where z_γ is the $100(1 - \gamma)$ percentage point of the standard normal distribution (easily obtained from tables). If use is lognormally distributed, a similar formula can be used since, then, log use is normally distributed. Thus, if log use is normally distributed with mean α and standard deviation β, we take

$$\log S = \alpha + z_\gamma \beta;$$
$$\text{i.e., } S = \text{antilog}(\alpha + z_\gamma \beta).$$

We can, of course, take logarithms to any base, provided the antilogarithm is taken to the same base. The mean and standard deviation of use, or log use, can be estimated from a large body of data in the usual way; these estimates are then used instead of the true parameters in the above equations.

Pegels suggests that a histogram of blood use be drawn and visually inspected; if it is skewed to the right, a lognormal distribution should be assumed; otherwise a normal distribution should be assumed. He gives an example in which the mean daily use is 5.73 units, with a standard deviation of 2.08 units. Then if we let $\gamma = 0.1$, we have

$$S = 5.73 + (1.28)\ (2.08) = 8.4,$$

and so we take an inventory level of nine units (1.28 is the $100(1 - 0.1) = 90\%$ point of the standard normal distribution). For the same set of data he calculates, under the assumption of a lognormal distribution, $S = 8.8$; so in this case the two assumptions lead to the same inventory level of nine units. It must be stressed that this example is a realistic one for the purposes of illustrating this method, but an unrealistic one from the point of view of many hospital blood banks. If we ignore the difficulty of obtaining an immediate delivery of ordered blood, the method is reasonable in the case of this example because (a) the mean daily use is relatively large, and in such cases its distribution can be well approximated by a continuous distribution such as the normal distribution, and (b) γ is fairly large, and in such cases the result obtained is less influenced by the particular form of use distribution assumed. It is the very features of the example that make the method reasonable that make the example itself unrealistic. In practice it is quite unacceptable to be short of blood of each type one day in ten; and, in fact, when the daily use of any type is greater than three or four units there should be no difficulty finding by trial and error an inventory level that leads to virtually no outdating or excess use.[36]

Although he points out that basically the same approach can be used if the distribution of blood use is a Poisson or negative binomial variable, Pegels considers this less desirable since the calculation of S is then no longer simplified by the existence of tables analogous to the percentage points of the normal distribution. In fact, he criticizes the assumption of a negative binomial distribution because of the limitations of its parameters. He incorrectly states that the parameter k must be integral which, if it were true, would certainly be a severe limitation. Using the usual definition[10] $x! = \Gamma(x+1)$, where Γ is the ordinary gamma function, it is clear that it is not necessary for k to be integral in the formula given earlier for the negative binomial distribution. The other limitation he points out, namely that the variance of a negative binomial distribution must always be greater than its mean, is not of practical importance; for, by its very nature, the distribution of blood use always has a variance greater than the mean. This arises because the daily use is the sum of "clusters" of blood units, each cluster being the number of units required for a particular patient. Provided (1) these clusters vary in size in a manner that is not dependent on the other cluster sizes needed during the same day, and (2) the daily number of patients of a particular blood type follow a Poisson distribution (which is certainly a close approximation to reality), the variance of the use distribution must necessarily be greater than the mean. (For this reason it is fairly safe to assume that the example given, in which the mean blood use is 5.73 and the variance is $2.08^2 = 4.33$, is based on artificial data.)

Turning now to the case where there is random input, it is a simple matter to use exactly the same approach by assuming the difference between use and input to be either normally or lognormally distributed, which will be the case if both use and input are either normally or lognormally distributed. We now determine S from the mean and variance of the distribution of these *differences,* again using the appropriate percentage point of the standard normal distribution. If we make the more realistic assumption, however, that both random input and use follow negative binomial distributions, an analytic solution is much more difficult; Pegels briefly indicates, in an implicit form, an approximate solution to this problem.

Another problem tackled is that of considering the blood bank as a whole, comprising eight "sub-banks" corresponding to the eight major blood types. If the overall allowable probability of excess use, for one or more blood types, is Γ, and the allowable probability of excess use for the i-th blood type is γ_i, then we have

$$\Gamma = 1 - \prod_{i=1}^{8} (1 - \gamma_i).$$

Given Γ there are many possible solutions for the γ_i, depending upon their relative sizes. If we wish all the γ_i to be equal, then the solution is simply

$$\gamma_i = 1 - (1 - \Gamma)^{1/8}.$$

Also considering the blood bank as a whole, we may wish to determine an inventory policy in which it is only possible to order blood of unspecified type. This situation is not realistic for a hospital blood bank that places its orders with a central blood bank, but does represent the situation of a central blood bank facing the problem of how many donors should be recruited each day. Suppose, for the i-th blood type, use is normally distributed with mean μ_i and standard deviation σ_i. This is a reasonable assumption for a large central blood bank, "use" now being taken to mean the dispatching of blood to other banks. Suppose we can draw blood from a large pool of donors in which a proportion p_i have blood type i. Given the bank contains x_i units of the i-th blood type, how much blood should we draw altogether (i.e., from how many donors should we draw blood), if we want the probability of use being in excess for the i-th blood type to be just γ_i? As before, we implicitly assume an immediate delivery of blood into the bank; so in practice we shall underestimate the amount we require. If we draw R units of blood, then the number of units of type i that will enter the bank is a random quantity that is approximately normally distributed with mean Rp_i and standard deviation $[Rp_i\ (1-p_i)]^{1/2}$. Taking the use distribution to be normally distributed with mean μ_i and standard deviation σ_i, the difference between use and input for the i-th blood type, if we draw from R donors altogether, is normally distributed with mean $(\mu_i - Rp_i)$ and standard deviation $[\sigma_i^2 + Rp_i(1-p_i)]^{1/2}$. Thus we must choose R such that there is a probability γ_i that this difference should exceed x_i, i.e., we must have

$$x_i = (\mu_i - Rp_i) + z[\sigma_i^2 + Rp_i(1-p_i)]^{1/2},$$

where z is the $100(1-\gamma_i)$ percentage point of the standard normal distribution. Pegels suggests an iterative method for solving this equation, but it is not difficult to show that the analytical solution we desire is

$$R = \frac{1}{2p_i}\{2(\mu_i - x_i) + (1-p_i)z^2 + z[(1-p_i)^2z^2 + 4(1-p_i)(\mu_i - x_i) + 4\sigma^2]^{1/2}\}.$$

If the inventory level x_i should happen to be much larger than the mean use μ_i, then it may be that no real solution is possible. In such a situation, which is more hypothetical than realistic, we take $R = 0$ and, even so, the probability of use being in excess is less than γ_i.

Finally, Pegels also considers the situation in which only the overall use is known, rather than the use for each blood type separately. Since, however, a central bank does not issue blood of unspecified type, the data from which type specific use can be estimated are never really difficult to obtain. For this reason it does not seem appropriate to consider this problem here.

CONCLUSION

It is clear from the articles reviewed here that computer systems and mathematical methods can be a great help in maintaining appropriate blood bank inventories. At present only a fraction of the blood banking community is using any of the methods we have discussed; and, furthermore, it must be stressed that these very methods exploit only a fraction of the capabilities of present-day computer systems and mathematical methodology. It is, therefore, appropriate, in conclusion, to indicate the sequence of steps that should be followed by any blood bank system wishing to take advantage of these capabilities: the first steps will necessarily make use of our experience to date in the field; and the later steps, which at present we can only speculate upon, will make use of the fast-growing advances of modern computer technology.

The first step is to obtain crude estimates of the use and/or demand distributions for each blood type, and similarly for random input if it exists. Given these data, the various methods discussed in this article can be used, at least approximately, to arrive at a rational ordering policy. For this purpose we must consider, simultaneously, the probability that use and/or demand will be in excess and the probability of outdating. Most of the ordering policies that have been proposed consider only the first of these quantities, since, in a sense, it is the most important. This, however, can lead to unrealistically high orders, especially for rare blood types and, hence, to an excessive amount of outdating. The simplest method to obtain approximations to the optimal inventory levels when the daily use is three units or less is to use the tables given by Elston and Pickrel.[36 38] These tables are certainly not strictly applicable to all hospital blood banks, but they can serve as providing first approximations. The statistical features resulting from keeping such inventory levels, such as excess use and outdating, can then be monitored and appropriate changes in the inventory levels made. For example, if the chosen level is expected, on the basis of the tables, to result in excess use four days per year, then the probability of being short on one day of the first week is $\frac{1}{13}$ and the probability of being short on two days of the first week $(\frac{1}{13})^2$ or $\frac{1}{169}$. Thus, whereas one crisis in the first week may be disregarded, a second should be taken as a definite indication that the chosen level is too low. On the other hand, at least a month should be allowed to elapse before using the observed rate of outdating to decide whether or not the chosen level is too high; the expected rate of outdating given in the tables assumes that the bank is already in an equilibrium state with respect to the age distribution of the blood in it.

In order to use the tables, a loss function, in terms of excess use and outdating, must be specified. This is not to be considered as a disadvantage of the method, but as a necessary fact of life if we wish to balance, in some sense, the losses due to excess use and outdating. The loss function can be looked upon as an economic loss, in terms of dollars, if one wishes; but this is not necessary. The fact that we may be unwilling, or even unable to express in monetary units the loss incurred when a specified amount of excess use and outdating occurs does not imply that it is impossible to express an appropriate loss function. Jennings[39] suggests that an "indifference map" be drawn up in the form of contours on the shortage-outdating plane: each contour is a curved line along which there is no change in "undesirability," just as contours on a map are curves along which there is no change in height. Given such a set of contours and an operational curve such as Figure 1, it is a simple matter to choose that point on the operational curve that has least "undesirability." This, however, is mathematically equivalent to minimizing a certain loss function, there being a one-one relationship between each possible loss function and each possible indifference map.

The second step in the evolution of any blood bank inventory system of at least a moderate size should be the introduction of a computer to keep records. How this is done will vary from institution to institution, and two examples of this have been discussed. It should be pointed out here that introducing a key punch into the blood bank is not the only method of doing this. Typewriter consoles linked to a remote computer is one possibility, and a further possibility, that does not appear to have been used yet in blood banking, is to have all transactions recorded in ordinary pen or pencil on special ledger sheets that can be optically read, and so directly entered into a computer without the possibility of any transcribing errors. Computer programs such as developed by Stewart and Stewart (1969) or Bove and McKay (1969) automatically collect the data necessary to maintain a rational ordering policy. Such programs can easily be extended to collect the data necessary for the application of Markov chain methods, as described by Pegels and Jelmert (1969), and a program could be written to apply their method in an effort to obtain better inventory levels or issuing policies.

The final step would be to have a system of programs available that would not only collect data in the manner we have discussed, but also analyze it periodically in a more sophisticated manner and so come up with better ordering policies. Statistical methods could be used to predict trends in future use, and to use other ancillary information in deciding on the optimal ordering policy. A knowledge of the number of units in both the assigned and the unassigned inventories should be informative in deciding on the optimal order to place, and, since physicians differ with respect to what may be termed their demand/use ratio (i.e., the ratio of their average demands to their average eventual uses of blood), a knowledge of the physicians for whom units have been assigned may well also be useful. It is conceivable that every blood bank could have its own simulation program, simulating its particular operation in some detail, and, thus, every day determine by exploratory simulation the best order to place for each blood type. At present this would probably be too costly of computer time, but in the future this may not be so. In any case it would always be better to refine the analytical methods as much as possible and only use simulations as a last resort. Finally, just as the major airlines have computer terminals in many airports to help them in the reservation of passenger seats, so it should be possible to link up all the blood banks of a region in one computer system. With the development of an appropriate program package, a computer at one blood bank should be able to "talk" to other computers at nearby banks and so be in a position to advise, for example, on the best way of obtaining a given amount of blood of a specified type in a hurry—so many units from specified professional donors, so many from bank A, so many from bank B, etc. taking into account the possible needs of all the banks in the system. We are entering the computer revolution, and there is no doubt that its impact in the area of blood bank inventory control will be as great as its impact in all other areas of our lives.

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