Statistical Analysis for Genetic Epidemiology (S.A.G.E.) Version 6.4.2 User Reference Manual

Department of Epidemiology and Biostatistics Wolstein Research Building 2103 Cornell Rd Case Western Reserve University Cleveland, Ohio 44106-7281

January, 2021

S.A.G.E. is an open-source software available through our web site: http://darwin.cwru.edu

NOTICE

The recommended way of referencing the current release of the S.A.G.E. programs is as follows:

S.A.G.E. 6.4.2 [2021]. Statistical Analysis for Genetic Epidemiology http://darwin.cwru.edu.

You are requested to send bibliographic information to opensagecwru@gmail.com about every paper in which S.A.G.E. is used (author(s), title, journal, volume and page numbers) to be posted in https://github.com/elstonsage/SageCore/wiki/Publication, where you can find a list of papers by other users of S.A.G.E. that we know of.

Contents

¹Covariates are pair-specific and are allowed only in the one-parameter model.

Chapter 1

Introduction

Statistical Analysis for Genetic Epidemiology (S.A.G.E.) is a collection of freely available compiled C++ programs that perform a wide variety of genetic analyses on both family data and data on unrelated individuals. The range of functionality includes tools for

- extracting summary statistics describing the data and evaluating general data quality,
- estimating allele frequencies and testing Hardy-Weinberg proportions,
- estimating heritability and familial correlations,
- inferring mixture models for genetic transmission and penetrance functions, including variable age of onset,
- estimating identity-by-descent (IBD) allele sharing probabilities between relative pairs,
- performing model-based linkage analysis,
- performing model-free linkage analysis,
- performing transmission/disequilibrium (TDT) analysis, and
- analyzing trait/allele associations in both family data and unrelated individuals.

S.A.G.E. runs on a variety of platforms: Linux, Windows, Solaris and Mac/OSX. The programs may be run either from a command line or from a cross-platform graphical user interface (GUI) that is included as part of the complete package. The software is extremely flexible with respect to the structure of input data files and, unless otherwise stated, the dependent traits may be discrete (including dichotomous data) or quantitative.

Users may download the current version of S.A.G.E. at any time from our web site, and please check for the most up-to-date information on the current version of S.A.G.E. programs at the following URL:

http://darwin.cwru.edu.

1.1 Program Descriptions

1.1.1 Summary Statistics

PEDINFO

PEDigree INFOrmation and statistics : Provides many useful descriptive statistics on pedigree data including means, variances and histograms of family, sibship and pedigree sizes, and counts of each type of relative pair.

1.1.2 Data Quality

MARKERINFO

MARKER INFOrmation: Detects Mendelian inconsistencies of markers in pedigree data.

RELTEST

RELationship TESTing : Indicates pairs of relatives to be reclassified according to their true relationship using multi-point genome scan data. The method is based on a Markov process model of identity-by-descent (IBD) allele-sharing along chromosomes. This program currently analyzes four different types of putative pairs: full sib pairs, half sib pairs, parent offspring pairs and unrelated marital pairs. A summary file is produced that contains the pairs to be reclassified together with their Mean Allele-Sharing Statistic, Parent Offspring Statistic and, for each individual, the percentage of marker data that is missing.

1.1.3 Allele Frequency Estimation

FREQ

Allele FREQuency estimator : Estimates founder allele frequencies and the inbreeding coefficient from each of a set of markers on related individuals with known pedigree structure, and in the case of co-dominant markers generates marker locus description files, needed by GENIBD, MLOD, and other S.A.G.E. programs.

1.1.4 Familial Aggregation

ASSOC

Marker-Trait ASSOCiations in Pedigree Data : Simultaneously estimates and tests from pedigree data the association between a trait and covariates, which can include marker phenotypes (e.g. for genomewide association,GWAS) that have been transformed into quantitative covariates, and residual familial correlations/heritability.

FCOR

Family CORrelations : Calculates multivariate familial correlations with their asymptotic standard errors. Calculates familial correlations for all pair types available in the pedigrees without assuming multivariate normality of the traits across family members. This program can also provide output that can be used by the GMDR Utility in the S.A.G.E. GUI to provide input for the program GMDR (not part of S.A.G.E.) that performs Multifactor Dimensionality Reduction.

1.1.5 Commingling Analysis

SEGREG

SEGREGation models : This program can be used to fit mixtures of two or three normal distributions, simultaneously applying a power transformation to the data and also allowing for both ascertainment and residual familial correlations.

1.1.6 Segregation Analysis

SEGREG

SEGREGation models : Fits and tests Mendelian segregation models in the presence of residual familial correlations. The trait analyzed can be quantitative, binary, or a binary disease trait with variable age of onset. This program can also be used for commingling analysis, to predict the major genotype of any pedigree member, and to prepare penetrance files for model-based linkage analysis.

1.1.7 IBD Allele Sharing Analysis

GENIBD

GENerate IBDsharing probabilities: Generates both single- and multi-marker identityby-descent (IBD) distributions using a variety of algorithms tuned for different types of relative pairs in pedigrees. Exact methods can be used for small pedigrees with loops, and a Monte Carlo Method is available for large extended pedigrees with loops. In the case of small pedigrees, IBD sharing can also be interpolated between markers.

1.1.8 Model-Based Linkage Analysis

LODLINK

Single-marker model-based LOD score LINKage analysis : LOD scores and recombination fractions are obtained between a marker and trait that follows any Mendelian model allowed by SEGREG (which can be used to generate the appropriate penetrance files). Test of linkage heterogeneity, and of linkage in the presence of linkage heterogeneity, are included.

MLOD

Multi-point model-based LOD score analysis : Performs multi-marker model-based LOD-score linkage analysis on small pedigrees. Analysis is greatly optimized for examining multiple one-locus trait models and will, in future versions, allow for meiosis specific (e.g., age and sex specific) recombination fractions.

1.1.9 Model-Free Linkage Analysis

LODPAL

LOD score Pair AnaLysis : Performs analysis based on the LOD score formulation for affected-sib-pairs (ASP). The current implementation is of the general conditional logistic model, including the one-parameter model that allows for the inclusion of all affected-relative-pairs, covariates and epistatic interactions. Alternatively, the oneparameter model can incorporate unaffected and discordant pairs.

SIBPAL

SIBling Pair AnaLysis : Performs mean tests, proportion tests for affected, discordant and unaffected sib pairs, and linear regression-based modeling of a weighted average of squared sib-pair differences and squared mean-corrected sums of a trait as a function of marker allele identity-by-descent sharing. Available analyses can use either single- or multi-marker IBD information, and models allow for both binary and quantitative traits due to multiple genetic loci, including epistatic interactions and pair-specific covariate effects.

RELPAL

RELative Pair AnaLysis: Performs a regression-based univariate or multivariate modelfree two-level Haseman-Elston linkage analysis that models trait data from relative pairs as a function of marker allele sharing identity-by-descent (IBD), as proposed by Wang and Elston (2005, 2006). Available analyses can use both single- and multipoint IBD information, and models allow for both binary and quantitative traits caused by segregation at multiple genetic loci, including epistatic interactions and covariate effects.

AGEON

AGE of ONset : Produces maximum likelihood estimates of the parameters of a mixed power-normal distribution for a binary trait with variable age of onset. The mean, variance and susceptibility parameters can be specified as dependent on covariates. These estimates are then used to produce two new quantative traits, trait susceptability and an age of onset residual, that can then be used in model-free linkage analyses.

1.1.10 Transmission Disequilibrium

TDTEX

Transmission Disequilibrium Test (EXact) : This program implements several asymptotic and exact versions of the transmission disequilibrium test (TDT) for testing linkage between marker and disease loci in the presence of allelic association. The exact tests are useful in cases where little data are available or there are many alleles at the marker locus. Different types of tests are available, including an exact test and a Markov chain Monte Carlo randomization test, as well as several exact marginal homogeneity tests.

1.1.11 Allelic Association

ASSOC

Marker-Trait ASSOCiations in Pedigree Data : Analyzes in the presence of familial correlations the association between trait (binary or quantitative) and covariates, which can include marker phenotypes that have been transformed into quantitative covariates, from pedigree data and/or unrelated individuals. Together with the *Transmitted Allele Indicator* (available as a user-defined function), performs a pedigree transmission disequilibrium test (TDT). This program will also estimate heritability and environmental familial correlations.

1.1.12 Haplotype Analysis

DECIPHER

Obtains maximum likelihood estimates of population haplotype frequencies for autosomal or X-linked markers, and determines all possible diplotypes and the most likely diplotypes for each individual. Estimates haplotype frequencies for different populations, as specified by the user, and performs likelihood ratio tests and permutation tests to compare haplotype frequency distributions famong populations.

1.1.13 Study Design

DESPAIR

Determines optimal two-stage linkage study design for affected relative pairs. Determines the minimally sufficient number of concordant and/or discordant pairs, and also the number of equally spaced markers, needed for the initial phase of a proposed linkage study.

1.1.14 SNP Marker filtering and SNP sequence analysis

SNPCLIP

Filters SNP markers on the basis of their minor allele frequencies, consecutive correlations, map locations and missingness. Within SNPCLIP is MUGS (Maximum Unbroken Genotypic Sequence), an analysis that aims to find maximum length common haplotypes in a group of individuals, or in each of two groups of individuals (e.g. affected and unaffected). This program is only accessible from the GUI and is described in the S.A.G.E. GUI Manual.

1.2 Program Limitations

All programs currently make the following assumptions in all of their analysis methods:

- 1. each genetic marker has a known genotype-phenotype relation (which may be either deterministic or probabilistic),
- 2. the founders of each constituent pedigree^{[1](#page-21-2)} are not inbred^{[2](#page-21-3)} and are unrelated to one another, and the pedigrees do not contain loops (except that MLOD can analyze small peigrees that have loops).
- 3. the members of each constituent pedigree are unrelated to the members of any other constituent pedigree.

1.3 Conventions Used in this Manual

This document uses the following typographical conventions to help clarify the correct specification of S.A.G.E. program commands and options:

- 1. All references to parameters and attributes are printed using a non-proportional font.
- 2. All references to *named constant* values (e.g., true and false) outside of a syntax table (see [3.2.4\)](#page-36-0) are printed using a bold font.
- 3. Examples of parameter files are printed using a non-proportional font.
- 4. Examples of program outputs are printed using a non-proportional font.
- 5. Technical terms that have not been previously introduced in the manual are printed using an *italics* font. The term's definition will be explicitly given if its meaning is not evident from the context.
- 6. Text that needs to be otherwise EMPHASIZED is printed using an UPPER CASE font.

¹The user defines pedigrees by giving each pedigree a unique identification number. A subset of pedigree members for whom there is no information on how they are related to other members of that pedigree is called a *constituent pedigree* and is treated as an independent pedigree in all analyses. With the exception of the program PEDINFO, the term *pedigree* will always refer to a *constituent pedigree* as defined here.

²Several programs allow for non-Hardy-Weinberg equilibrium proportions and the SEGREG program allows for general transmission models.

Chapter 2

Running S.A.G.E. Programs

S.A.G.E. programs may be executed from the provided Graphical User Interface (GUI) or, alternatively, by means of a command line directive that specifies the name of a selected program followed by a list of *arguments* to the program^{[1](#page-22-3)}.

2.1 Graphical User Interface (GUI)

Please refer to the S.A.G.E. GUI user manual for how to run S.A.G.E. programs using the Graphical User Interface (GUI).

2.2 Command Line

Note that, whereas every program described in this manual (except DESPAIR, which is a web-based interactive program) can be run using the GUI, there are some programs that can only be run using the GUI. See the S.A.G.E. GUI user manual for these programs.

A S.A.G.E. program is run by specifying the name of the program followed by the input files on the command line. The input files can be listed in two different ways, one with flags and another without.

With flags, the syntax requires the user to precede each filename with a flag indicating the filetype as follows. Order does not matter if flags are used.

	Flag Filetype	Description
$-p$	parameter	Parameter File
-d	pedigree	Pedigree Data File

 1 The GUI is designed to generate syntactically correct lists of program arguments based on the user selections in the various screens and dialogs. The argument lists are in turn forwarded to the desired S.A.G.E. program(s) for processing. Although the GUI does perform validation of user selections before submitting them for processing, it may nevertheless be possible for the user to generate an invalid S.A.G.E. command through the GUI. The user is therefore advised to check the information file (*.inf) as well as the standard S.A.G.E. outputs to the console when the program does not seem to work as expected.

In the absense of flags, filenames must be given in the specified order.

The program specific usage information can be displayed by entering an application name or an application name followed by -h or -? at the command prompt. For example, entering "decipher -h" will result in the following.

```
usage (without flags): decipher <parameters> <pedigree> [locus]
usage (with flags): decipher <-p parameters> <-d pedigree> [-l locus] [-g genome]
Command line parameters:
parameters - Parameter File
pedigree - Pedigree Data File
locus - Locus Description File (optional)
genome - Genome Description File (optional)
```
Items in angled brackets (<>) are required. Those in square brackets ([]) are optional. Note that in this example the genome file is not listed in the unflagged format. Therefore, a user wishing to use a genome file MUST use the flagged format.

Here is a second example.

```
usage (without flags): sibpal <parameters> <pedigree> <ibd> ...
usage (with flags): sibpal <-p parameters> <-d pedigree> <-i ibd> ...
Command line parameters:
parameters - Parameter File
pedigree - Pedigree Data File
ibd - IBD Sharing File
```
The ellipsis, (\ldots) , indicates that more than one ibd file may be supplied, for example "sibpal -p" my_par -d my_ped -i ibd1 -i ibd2 -i ibd3".

Multiple pedigree files may be supplied for any of the S.A.G.E. applications, in which case only the name of the first pedigree file is given on the command line. Pedigree blocks in the parameter file must be supplied for each pedigree file. If there are multiple files, file name must be part of these pedigree blocks.

A typical run of a S.A.G.E. program, for example FCOR, may look like the following:

```
>fcor -p data.par -d data.ped
FCOR -- xx Nov 201x hh:mm:ss -- [S.A.G.E. v6.x.0; xx Nov 201x]
Reading Parameter File.............................done.
Reading pedigree file.....................
               from data.ped..................done.
Sorting pedigrees.................................done.
.
.
.
Analysis complete!
```
Chapter 3

Program Input and Output

Each S.A.G.E. program requires several input files in order to run. No program requires all of the possible input files. Refer to the individual program documentation for specific information on which files are required. The file types currently used for program input are:

*^a*This file, as well as the genome description file, can be produced using the graphical user interface (GUI).

*b*Single-marker in the sense that information is used from only one observed marker locus at a time. When performing linkage analysis this is often called "two-point" analysis.

Each program also produces one or more output files that contain results and diagnostic information. Refer to the individual program documentation for specific information on which files are produced and details of what they contain.

The file types currently used for program output are:

3.1 The Pedigree Data File

For family data to be accurately analyzed they must be described and represented precisely. The following are the definitions of various non-obvious family structures and relationships that are used throughout this manual^{[1](#page-26-2)}.

*^a*Some other software packages refer to our definition of pedigrees as kindreds.

b " ID" is an acronym for *identifier* , and is used frequently throughout this document.

*^c*Founders do not include singleton individuals.

*^d*A constituent pedigree is what is typically referred to as a pedigree in the literature. The distinction is made because of the prevalence of incomplete and fragmented datasets.

*^e*Singletons are sometimes not differentiated from founders in the literature.

3.1.1 Pedigree Data File Specification

A *pedigree data file* is a text file composed of one or more records, each of which contains infor-mation about a single individual. Each record must end with a carriage return or linefeed character^{[2](#page-26-8)}

 1 Some of these definitions are fairly technical but under most circumstances the conventional definitions will suffice.

²Any combination of carriage return and line feed characters is sufficient to terminate a record. This allows pedigree data files from most popular operating systems to be used without translation.

and contains the following fields:

- 1. If the Pedigree ID field is absent in the pedigree data file, all individuals are assumed to belong to the same pedigree.
- 2. Implicit in this is the possibility that the same Individual ID may appear more than once in a given pedigree data file, referring to a different individual at each occurrence.
- 3. At the user's option, the pedigree data file may list the father's ID first, followed by the mother's ID, or vice versa. Also note that partial lineage, i.e. only one parent specified, is not allowed. If Parent ID fields are absent in the pedigree data file, all individuals are treated as singletons unless there exists a Pedigree ID field and the user chooses to treat the individuals sharing the same Pedigree ID as sibs, by including the treat_as_sibs parameter in the parameter file (see [3.2.5.2\)](#page-42-0).
- 4. Incorrect use of the word *gender* is studiously avoided here. As the poet says, "Nouns have gender, whereas people have sex ... and enjoy it!" If the Sex field is absent while the Parent ID fileds exist in the pedigree data file, the user is required to explicitly acknowledge this situation by including the no_sex_field parameter in the parameter file (see [3.2.5.2\)](#page-42-0) to proceed with any analyses. However, the analyses may produce unpredictable results.
- 5. Even though some fields are not required to be included in the pedigree data file, they are requred for many analyses to be meaningful. For eaxample, genotypic data are required for programs that perform linkage analysis, allelic association analysis, etc.

The individual fields in a record for an individual are separated by one or more characters, known as *delimiters* , which are usually not present in any of the data elements themselves. Commonly

used delimiters are the comma, the tab, and the space, but any non-alphanumeric character may be used. If your data are separated by a fixed known delimiter, then S.A.G.E. will read your pedigree file as character delimited records, and you will need to specify which delimiter is used along with some additional *metadata*^{[3](#page-28-0)} that specify the names and types of the fields in your pedigree records.

Files that are formatted for LINKAGE, GENEHUNTER, PAP, GAS or similar computer programs may all be read as character delimited records with little or no modification^{[4](#page-28-1)}. Programs that readily generate data in a character delimited form are spreadsheet programs like Microsoft Excel, most pedigree drawing programs, and most database programs. Microsoft Excel files can be used to run S.A.G.E. using the GUI. Please refer to the GUI user manual.

THE FORMAT OF THE CHARACTER DELIMITED DATA FILE IS DEFINED BY A CHARACTER DE-LIMITED LIST OF DISTINCT NAMES THAT IDENTIFY EACH FIELD. THIS LIST OF NAMES MAY BE SPECIFIED AS THE FIRST LINE, OR HEADER, OF A CHARACTER DELIMITED DATA FILE; OR, ALTERNATIVELY, IT MAY BE GIVEN AS A SET OF PARAMETERS IN A PEDIGREE BLOCK WITHIN THE PARAMETER FILE.

The name of each field in this list has no default semantic meaning, and the field it identifies may be used for any purpose once read in. Associating a field with a meaning, such as a pedigree ID, individual ID, marker phenotype, trait, etc. is accomplished by specifying parameters and attributes in the pedigree block (see [3.2.5\)](#page-38-0) of the parameter file to map the field names to data field types. It is not necessary to specify all fields that exist in the pedigree data file in the pedigree block of the parameter file, but it is important that no field in the pedigree data file be used as a parameter value more than once. Whitespace is stripped from the beginning and end of the content of each field.

Several options are provided to let the user modify the way a character delimited pedigree data file is processed by S.A.G.E. programs. The sets of characters that represent whitespace and delimiter characters may be redefined. There is an option that alters the way multiple consecutive delimiter characters are interpreted, by treating them as a single delimiter. This is extremely useful when reading multiple space delimited, or other fixed column formats, that do not include empty fields. Empty fields are a problem in this mode because it is not possible to detect them. For example, suppose each line in the following fixed column, space delimited, file is parsed into 6 fields using the delimiters and delimiter_mode options to read multiple blanks as a single field delimiter and skip leading and trailing blanks. The following delimited pedigree file is correctly specified:

If 0, the missing value code for parents and traits in this example, were replaced with a space character as indicated below, the resulting fixed column records would be parsed inconsistently. The two parents would have the SEX field as their MOM field, as well as other errors due to missing values not being detected.

³Database terminology that means "information about the data", i.e., field names, data types, value ranges, etc.

⁴If necessary, column-delimited input files, such as those required for PAP can be imported into a spreadsheet program (Microsoft Excel, for example) and then exported in a character delimited format.

Here is a typical pedigree data file in comma delimited pedigree data file that includes the name of each field in a header line:

Suppose each record in the above data file is one line long and you want to use the following fields:

then the following pedigree block in the parameter file can be used to read this pedigree data file (in the following and elsewhere in this document, any line that starts with '#' is a comment line and is ignored by the program; see 3.2.2):

```
# Example - character delimited
pedigree
{
   # The following format string could be used if the pedigree file did not
   # already include a header line.Do NOT include both!
   # format="PID,ID,P1,P2,SEX,JUNK,D42S1,D42S2,D42S3,D42S4,D42S5,D42S6,TRAIT 1"
   pedigree_id=PID # Pedigree Field Specification
   individual_id=ID
   sex_field=SEX, male="m",female="f",missing="x"
   parent_id=P1
   parent_id=P2
   trait="TRAIT1",name="DBP",missing="XXXX" # order is irrelevant
   trait="CAT",categorical,values="RED,GREEN,BLUE"
   marker=D42S4
   marker=D42S6
   marker=D42S1
   marker=D42S2
   marker=D42S3
   # Pedigree encoding information:
   individual_missing_value="0"
}
```
3.1.2 Pedigree Data Quality

Users are always well-advised to ensure that their pedigree data files are as error-free as possible^{[5](#page-30-1)}, with particular attention paid to the correctness of family relationships within individual pedigrees. Nevertheless, S.A.G.E. programs are able to run in the presence of less-than-perfect data. Missing data will typically not prevent S.A.G.E. analyses from running to completion.

Note: *If the pedigree block of the parameter file lists a variable that does not appear (or is spelled differently) within the corresponding pedigree data file, S.A.G.E. will issue an appropriate error message and halt immediately.*

⁵A well-known software apothegm is *"garbage in, garbage out* ", also expressed as the acronym *GIGO* .

3.2 The Parameter File

User options for analysis are specified to S.A.G.E. programs as a list of instructions within a parameter file ^{[6](#page-31-2)}. When a particular S.A.G.E. program is executed it evaluates the contents of the specified parameter file to determine

- 1. how to interpret the contents of the given pedigree data file,
- 2. how many different analyses have been requested and
- 3. which options have been specified for each analysis.

A parameter file is simply a text file containing a list of S.A.G.E. program instructions written according to a specific syntax (see [3.2.2\)](#page-32-0). It commonly consists of four different types of instruction blocks; one or more pedigree blocks to determine how to read and interpret the contents of the given pedigree data file(s) (see [3.2.5\)](#page-38-0), one marker block to set overall options on how to read marker phenotype data (see [3.2.6\)](#page-54-0) if there exists marker phenotype data in the pedigree data file(s), zero or more function blocks to create new traits or covariates as a function of existing data (see [3.2.7\)](#page-57-0), and zero or more program-specific analysis blocks which include the program specific options (see the program-specific Input section).

A single parameter file may be used to specify options for one or more S.A.G.E. programs in any combination. In other words, one parameter file could specify analysis options for several different S.A.G.E. programs, or different options for repeated calls to the same program, or both. And, of course, the user always has the option of creating a set of different parameter files if that makes it easier to manage a given project^{[7](#page-31-3)}. Since the parameter file also contains user-supplied specifications on how to interpret the pedigree data file, the ability to specify an arbitrary set of S.A.G.E. analyses within a single parameter file makes the software very flexible.

3.2.1 Creating a Parameter File

One of the primary functions of the GUI is to create parameter files for S.A.G.E. programs. The GUI is designed to translate the user's selection on the various screens and dialogs into syntactically correct lists of program arguments, which are automatically passed into the appropriate S.A.G.E. program. This feature is intended to reduce the complexity associated with learning the syntax of S.A.G.E. parameter file, and is expected to be particularly beneficial to novice users of the software, who may initially skip many of the details in the rest of this chapter, but it is recommended that they read [3.2.7.](#page-57-0) A function tool is available in the GUI and can be used at any time after the pedigree file has been imported into a project. Experienced users who prefer to create and edit their parameter files directly continue to have the option of doing so.

⁶The reader is cautioned that the word *parameter* will have two meanings in this document. In one context it will refer to the set of defined S.A.G.E. *keywords* , but in a statistical context it will refer to some distribution characteristic (e.g., the mean (μ) or variance (σ^2) of a normal distribution). One goal of the typographical conventions (see 1.3) is to make the context of this word as clear as possible.

⁷S.A.G.E. programs accept only one parameter file at a time (the one named as a program argument), regardless of the number available.

A parameter file may be created and modified using a standard text editor on the local S.A.G.E. platform (i.e., the system on which S.A.G.E. has been installed), or the file may be produced on a different system and copied to the local S.A.G.E. platform. The user will normally want to copy the parameter file into the same directory that contains the pedigree data file for a given project, although this is not required.

In computing environments that include both Unix workstations and Windows PCs, many individuals find the text editors available in Windows to be more user-friendly and convenient, and therefore would prefer to edit their parameter files with either Notepad or WordPad *.* Users who take this approach must remember to remove the spurious *carriage return* character (^M) which appears at the end of each line of the text file after it has been copied to the Unix target directory^{[8](#page-32-1)}.

3.2.2 Parameter File Syntax and Structure

A parameter file consists of a list of S.A.G.E. program instructions known as *statements* . When a particular parameter file is passed to a S.A.G.E. program (as a command line argument), the specified program reads each line in the parameter file, from top to bottom, and configures itself to perform the analyses indicated by the listed statements.

All S.A.G.E. statements are formed according to the following format:

```
parameter [= value][, attribute [= value]]*
[{
   [statement]*
}]
```
in which the square brackets ([]) indicate groupings of optional terms and are not to be entered by the user. The asterisk (*) indicates that the preceding group or item may be repeated zero or more times. Note that the brackets $($ []) and asterisk $(*)$ are artifacts of the above format definition, and are not to be entered by the user.

In words, the above format definition says:

" A *statement* is a parameter followed by an optional equal-sign-and-value pair^{[9](#page-32-2)}, followed by zero or more optional comma-and-attribute pairs (in which each attribute may be followed by an optional equal-sign-and-value pair). This totality, in turn, is optionally followed by a brace-enclosed list of zero or more *statements*."

The terms parameter and attribute represent S.A.G.E. *keywords* specified throughout this document, and the braces ({ }) are used to enclose an optional *block* of zero or more subsequent statements. Further, the \lt and gt symbols may be used instead if there are no { and } symbols on the user's keyboard.

The recursive manner by which a statement is defined in terms of itself is no accident and is, in fact, a common way to specify a formal language structure. Dut to the recursive nature of their definition, statements can be *nested* when listed in the parameter file. That is, a particular statement may be

⁸Utility programs available under Unix, such as *dos2unix* make this task fairly easy.

⁹ Parameters and attributes often do not require an explicit value to be assigned, allowing the user to run the selected S.A.G.E. program with its default values.

specified as containing another *statement* which itself comprises one or more *statements*, etc. This manual refers to outermost *statements* containing one or more *statements* within a pair of braces as *blocks.* When a brace-enclosed block is nested within another enclosing block, the nested blocks are referred to as *sub-blocks* .

Some possible statement structures are:

```
Example 1: parameter
Example 2: parameter, attribute = value
Example 3: parameter = value
Example 4: parameter = value, attribute
Example 5: parameter = value, attribute = value, attribute = value
Example 6: parameter = value
           \mathcal{F}parameter = value
             parameter, attribute = value
           }
Example 7: parameter = value
           \sqrt{2}parameter = value
             parameter
              {
               parameter = value
               parameter, attribute = value
             }
           }
```
The S.A.G.E. statement grammar described above is complex, which can make the software difficult to learn. As noted previously, the S.A.G.E. GUI is designed to eliminate the burden of learning the parameter file syntax; however, there may be times when an experienced user would prefer to manipulate the parameter files directly.

The following provides clarifying details and examples of parameter file syntax.

- 1. The specific parameters and attributes listed within this document are S.A.G.E. *reserved words* , meaning that they must be spelled exactly as shown in their corresponding syntax table (see [3.2.4\)](#page-36-0).
- 2. The names of traits and covariates found in the pedigree data file may be the same as the names of parameters and attributes, although this practice is likely to cause confusion and is therefore not recommended.
- 3. White space, including blanks, tabs and newline characters, are required only to differentiate between successive parameters, attributes and values, and are otherwise ignored^{[10](#page-33-0)} by

¹⁰White space that occurs as part of a QUOTED character string (e.g., "Body Mass Index") is not ignored.

S.A.G.E. programs. They may usually be inserted or omitted from statements at the user's discretion.^{[11](#page-34-2)}

- 4. Blank lines between successive statements are ignored and may be inserted as necessary to make the parameter file easier to read.
- 5. A single statement may fit on a single line or may continue over several lines. Further, the placement of braces (for enclosed blocks and sub-blocks) is left entirely to the discretion of the user. The following example shows two ways to specify the same segreg statement:

```
segreg, out= "my_analysis.out"{trait= BMI type_mean{option= three}}
segreg, out = "my_analysis.out"{
   trait = BMI
   type_mean
   {
      option = three
   }
}
```
6. The insertion of a pound sign (#) at any point of a line in a parameter file causes S.A.G.E. to ignore the remainder of that line. Thus, the user can *comment* on the contents of a parameter file that may need to be reviewed at some future time. The following example shows how the above-listed segreg block might be commented:

```
# Perform analysis on Body Mass Index
#
segreg, out = "my_analysis.out"
{
   trait = BMI
   type_mean
   {
      option = three # Run the 3-mean model
   }
}
```
3.2.3 Parameter and Attribute Values

3.2.3.1 Character Strings

When a particular parameter or attribute takes a *character string*^{[12](#page-34-3)} for its value, the user should enter the desired *alphanumeric* character sequence ^{[13](#page-34-4)} after the equal sign (=). Enclosing *double quotes*^{[14](#page-34-5)} are only required in the following cases:

• the string contains blank spaces (e.g., "Alice in Wonderland"),

¹¹Many users find that judicious use of blank spaces can make a parameter file easier to read, and therefore less prone to error.

 $12A$ character string is simply a contiguous sequence of zero or more letters, digits, or other typographic symbols, including spaces.

¹³An alphanumeric string may contain only letters (upper or lower case) and decimal digits.

¹⁴We distinguish between two kinds of quotation marks: double quotes (" $"$ ") and single quotes ("). Unless otherwise stated, the double quotes should be used whenever the syntax rules call for a quoted string.

- the string contains *non-alphanumeric* characters^{[15](#page-35-1)} (e.g., "Alice-in-Wonderland")
- the string contains no characters at all i.e., it has length of zero (e.g., ""). A zero-length string is sometimes referred to as a *null* string.

S.A.G.E. is *case insensitive* with respect to the names of traits and covariates and, therefore, the following statements are equivalent:

 $\text{trait} = \text{HT}, \text{ type} = \text{continuous}$ $\text{trait} = hT$, type = continuous trait = Ht, type = continuous trait = ht, type = continuous

In all other cases, S.A.G.E. is *case sensitive* .

3.2.3.2 Numeric Values

When a particular parameter or attribute takes a numeric quantity for its value, the user is required to enter a constant according to the normal conventions of decimal notation. Specific constraints on the value are described as follows:

In addition to decimal quantities, S.A.G.E. also accepts the following *named constants* :

- pi, designating the transcendental number $\pi = 3.141592654...$
- e, designating the base of the natural logarithms $= 2.718281828459...$

¹⁵Typographic symbols OTHER than letters or digits: ~!@#\$%^&*()+'-={}|[]\:";'<>?,./
The following example shows some ways in which numeric values may appear within S.A.G.E. statements:

```
segreg
{
   composite_trait
   {
      covariate = BMI, val =27.69
   }
  transmission
   {
      option = homog_general
      tau =A*, val =0.5}
}
```
3.2.4 Reading and Interpreting the Syntax Tables

For every *statement* defined within the S.A.G.E. family of programs, this user document provides the following information in tabular form:

- parameter designation
- list of attributes associated with a given parameter
- a brief explanation
- range of valid or possible values that the parameter or attribute can take
- the default value
- whether or not the parameter or attribute is required
- a list of applicable notes when more detailed explanation is required; these notes will always be found immediately at the end of the table.

Note that the information on the range of valid or possible values and the default value is only relevant when a value is required for the given parameter or attribute.

To understand how to interpret the syntax tables used in this document, consider the following example:

^a The occurrence of multiple parameter names in a single cell means that the explanatory information at the right is applicable to all of them, and any attributes listed within the cell are also applicable to all of them.

^b An acronym for *identifier* .

^c For users who are accustomed to spreadsheets, the database term *field* is analogous to *column* , and the term *record* is analogous to *row* .

^d None means there is no default value specified.

e If Yes, then the listed parameter or attribute is required, and the user must explicitly enter the listed parameter or attribute into the parameter file, optionally followed by an assignment expression with a value. If No, then the listed parameter or attribute is NOT required because either it is an optional feature or the specified default value will be used in the analysis. Note: When relying on default values for a given analysis, the user should take care to ensure that they are appropriate for the intended model.

^f The applicable notes will be found immediately below the table.

^g Attributes are indented with respect to their associated parameters, but appear in the same cell. Relevant explanatory information appears to their immediate right.

^h N/A means *not applicable* , i.e., that the parameter or attribute in question is *self-defining* and does not take on any values.

3.2.5 The pedigree Block

A pedigree block determines how to read and interpret the contents of the given pedigree data file. Unless otherwise noted, all parameters and their corresponding attributes must be specified within a pedigree block of a parameter file which starts with a pedigree parameter.

The following table shows the syntax for a pedigree parameter:

Notes

1. S.A.G.E. programs are capable of processing multiple pedigree data files simultaneously. This feature is especially useful for analyzing marker data that span the entire genome, in which case each chromosome is normally allocated to its own data file. To analyze data across multiple pedigree files, create a separate pedigree block for each pedigree file, and use the file attribute to name a particular file, as in the following example:

```
pedigree, file = "Chr1.ped"
{
  delimiters = "\tt t" # The '\tt t' indicates the tab key
  delimiter_mode = multiple
  individual_missing_value = 0
  ...
  pedigree_id = PIDindividual_id = ID
  parent_id = P1
  parent_id = P2
  sex_field = sex...
  allele = D1S2195a, name = D1S2195
  allele = D1S2195b, name = D1S2195
  ...
}
pedigree, file = "Chr2.ped"
{
  delimiters = "\t"
  delimiter_mode = multiple
  individual_missing_value = 0
  ...
```

```
pedigree_id = PID
   individual_id = ID
  parent\_id = P1<br>parent\_id = P2
   parent_id = P2<br>sex_field = sexsex_field
   ...
   allele = D2S2195a, name = D2S2195
  allele = D2S2195b, name = D2S2195
   ...
}
pedigree, file = "Chr3.ped"
\mathfrak{c}delimiters = "\t"
  delimiter_model = multipleindividual_missing_value = 0
   ...
  pedigree_id = PID
   individual_id = ID<br>parent_id = P1
  parent_id
  parent_id = P2
  sex_field = sex
   ...
  allele = D3S2195a, name = D3S2195
  allele = D3S2195b, name = D3S2195
   ...
}
```
2. Even if the user specifies a file name at the start of each pedigree block, as shown in the above example, it is still necessary to supply the name of a pedigree data file on the program command line when running the program. The pedigree data file name specified at the S.A.G.E. program command line will automatically be assigned to the first pedigree block that does not specify the file attribute. Suppose the statements listed in the above example were contained in a parameter file named *hypertension_study.par* . Then the file attribute for the *first* pedigree block would be optional if the command line were:

```
>freq hypertension_study.par Chr1.ped
```
3.2.5.1 Parameters for General Pedigree File Formatting

The following table lists the parameter for the pedigree data file formatting that may occur in a pedigree block.

Notes

- 1. The format parameter is used to list the name of each field in the character delimited data file. Its value should be a delimited list of field names in the same order as those to be read from the file. The delimiter characters that separate each field name in this list are the same as those given in the delimiters parameter. If this parameter is not given, or is empty, then the first line of the character delimited pedigree file will be used to specify the format parameter.
- 2. The delimiters parameter specifies the characters that separate fields in each record. As a result, the delimiter characters should not be present in any fields. The default is that any comma (,) or tab (\t) character is interpreted as a delimiter character. Similarly, the whitespace parameter specifies characters that will be ignored when they appear at the beginning or end of fields.
- 3. The delimiter_mode parameter is used to alter how records are parsed. When the value of delimiter_mode is set to single each delimiter character found will terminate the current field. When the value of delimiter_mode is set to multiple, consecutive delimiters are treated as a single delimiter and delimiters that occur at the beginning and end of the record are ignored. Typically, tab and comma delimited files should be set to the value single, while space delimited files should be set to the value multiple.
- 4. By default, each individual in a pedigree must have one record in the pedigree data file. However, data on sibships without parent data are not uncommon. Distinguishing parent IDs must still be assigned to all individuals, but empty records for the dummy parents can be omitted if the require_record parameter is set to false.

3.2.5.2 Parameters for Individual and Family Identification Fields

The following table lists the parameters and attributes for individual and family identification fields in the pedigree data file that may occur in a pedigree block.

Notes

- 1. When the pedigree_id parameter is specified, 3 new traits (FAMILIAL_INDICATOR, FOUNDER_INDICATOR, PEDIGREE_SIZE) are automatically created for each individual with values as follows:
	- FAMILIAL_INDICATOR
		- \int = 1 if an individual belongs to a pedigree of size > 1
		- $= 0$ otherwise
	- FOUNDER_INDICATOR

 $= 1$ if an individual has both parents missing

-
-
- PEDIGREE_SIZE the number of individuals in the pedigree to which an individual belongs
- 2. The parent_id parameter is typically specified twice, once for each parent. If the parent_id parameters are not specified because they are absent in the pedigree data file, all individuals are treated as singletons unless there exists a Pedigree ID field specified with a pedigree_id parameter and the user chooses to treat the individuals sharing the same Pedigree ID as sibs by including the treat_as_sibs parameter in the pedigree block.
- 3. This is the code that is used in the Parent ID fields of founders.
- 4. When the sex_field parameter is specfied, a new trait called SEX_CODE is automatically created for each individual with values as follows:
	- male $= 0$
	- female $= 1$
- 5. If sex_field is absent while parent_id fields exist in the pedigree data file, the user is required to explicitly acknowledge this situation by including the no_sex_field parameter in the pedigree block.
- 6. Many S.A.G.E. algorithms rely on sex for structural information. Even when sex does not affect the outcome of an analysis, missing sex information may cause the program to behave improperly or even crash. If your analyses involve pedigree structure, this should be corrected before continuing. If you would like to continue the analysis without correcting this issue, you must place a no_sex_ok parameter in the pedigree block.

3.2.5.3 Parameters for Trait and Covariate Fields

The following table lists the parameters and attributes for trait and covariate fields in the pedigree data file that may occur in a pedigree block.

Notes

- 1. The trait and covariate parameters all perform the same basic function; the values assigned to them identify fields in the pedigree data file that contain quantitative or discrete phenotypic information The following guidelines may clarify when each different parameter should be used:
	- (a) covariate fields are a generic designation and convey no suggestion of how the field is to be used.
	- (b) trait fields are selected by many (but not all) analyses to be automatically used as major (dependent) variates.

Thus these parameters simply provide hints to S.A.G.E. on how to make reasonable use of phenotypic information. Refer to the program specific documentation for information on the specific behaviors of these parameters and how to override them.

Each trait field in a record may contain any character string that represents

- a missing value code,
- the affected or unaffected trait code (for a binary trait), or
- a numeric value (for a quantitative trait).
- 2. The trait and covariate parameters should be included for each field in the pedigree data file that contains quantitative or categorical phenotypic information. The value of each such parameter should be set to the name by which it will be referred to in the rest of the parameter file and in the program output. Remember: any field specified as a trait will automatically be analyzed by some S.A.G.E. programs, whereas fields specified as covariate will be analyzed optionally, depending on whether or not they have been listed within the relevant analysis block.
- 3. If the parameter file indicates that a trait be interpreted as categorical, then S.A.G.E will interpret each unique alphanumeric trait value as a distinct categorical code. For instance, if the trait in question has values "red", "green", and "blue", then S.A.G.E. will automatically recognize those alphanumeric strings as unique categorical values. Please note that S.A.G.E. examines the entire alphanumeric string to determine its uniqueness. If a trait is specified as categorical, then values "1" and "1.000" will be understood as two different categorical values.
- 4. If a trait is specified as categorical, the user can (optionally) include the values attribute. This attribute takes the form of a comma-delimited list of alphanumeric strings. Each string is understood to be a unique and valid categorical value. When reading in the pedigree data,

if this attribute is included, S.A.G.E. will consider invalid any categorical value it encounters that is not already in the values list. For instance, values = "red, green, blue" would limit allowable categorical values to those three strings. An individual with the value "redd" or " green" would be considered missing for the trait in question.

5. A name attribute may optionally be specified for trait and covariate parameters. If a name attribute is not specified, the trait name is assumed to be the field name. This feature is useful when the field names listed in the pedigree data file are obscure or unclear (usually due to abbreviation), and the user would like to create analyses, models and reports with more informative names. If a name attribute is specified, this user specified trait name should be used in the analysis blocks and the name will be used in all output files.

For example, if the pedigree data files contains four fields named Trait1, Trait2, Covariate1, and Affection, then the user may specify alternate names as in the following example:

```
covariate = Trait1, name = "Generic trait", missing = "X"
\text{trait} = Trait2, missing = "-99"
covariate = Covariate1, name = "Covariate #1"
trait = Affection, binary, affected = 1, unaffected = 0, missing = "?"
```
As a result, the field originally designated as "Trait1" should be referenced as "Generic trait" within S.A.G.E. analyses, and similarly, the field originally designated as "Covariate1" should be referenced as "Covariate #1" within subsequent analyses.

- 6. The covariate_list parameter specifies multiple covariates simultaneously. This is prefered over specifying covariates individually when covariates all have the same formatting and there are many covariates in the pedigree data file. The covariate_list parameter has the following attributes, corresponding to the same attributes as used for specifying covariates individually except name attribute:
	- missing
	- binary
	- affected
	- unaffected
	- categorical
	- values
- 7. The start attribute indicates the first covariate in the set, and the end attribute specifies the last. The start and end attributes are required if the covariate_list parameter is specified, and their omission prevents parsing the covariate list (a fatal error is generated). Both the start covariate and end covariate must be present in the pedigree data file, with end following start. A fatal error will result if this is not the case. A covariate_list with start and end being the same value will be interpreted as a single covariate.
- 8. Multiple covariate lists can be specified, but they may not overlap. Similarly they may not overlap with any other fields. When multiple covariate lists and fields are not disjoint, a fatal error is issued.
- 9. Users may set the missing option for a string parameter just as with the trait and covariate parameters.

3.2.5.4 Parameters for Genotype Data Fields

The following table lists the parameters and attributes for genotype data fields in the pedigree data file that may occur in a pedigree block.

Notes

- 1. For each locus, the information can be modified by adding the proper attributes to marker or allele parameter within the pedigree block. Each marker parameter and associated attributes in the pedigree block specifies the infomation relevant to that specific marker locus only while the information on marker block (see [3.2.6\)](#page-54-0) applies to all marker loci.
- 2. The values assigned to them identify fields in the pedigree data file that contain allele or marker phenotipic information.

Each allele field in a record may be any character string that represents:

- a missing value code, or
- a single allele name.

Each marker field in a record may be any character string that represents:

- a missing value code,
- an allele name, followed by the allele delimiter character and another allele name, or
- a marker phenotype name.
- 3. A single marker parameter or two allele parameters should be included for each marker locus in the pedigree data file. Each marker locus field that is to be used should have a corresponding entry in the marker locus description file that defines its alleles, genotypes and phenotype to genotype mapping. THOSE NOT FOUND IN THE MARKER LOCUS DESCRIP-TION FILE WILL NOT BE ANALYZED BY ANY APPLICATION THAT REQUIRES THE MARKER LOCUS DESCRIPTION FILE. (Provided that the markers are codominant, the marker locus description file can be generated automatically by the FREQ program, so in general this requirement should not pose a great problem to the user. See [3.3](#page-71-0) for markers that are not codominant.)
- 4. E.g., to specify three markers named "D42S1", "D42S2", "D42S3", a trait named "Trait1", a trait-marker^{[16](#page-51-0)} called "MOD", and a binary covariate named "Cov"; marker D42S1 is given by two allele fields, and the others are marker genotype fields:

¹⁶ A *trait-marker* is simply an observable biological trait, such as blood type or an enzyme activity level, that is used in a model-based linkage analysis. We use the term *trait-marker* because the trait is used in the analysis like any other marker phenotype (eg., SNP or microsatellite phenotype) but *without* any assumption of codominance.

```
# Order is irrelevant
allele = D42S1a, name = D42S1 # First allele of D42S1
allele = D42S1b, name = D42S1 # Second allele of D42S1
marker = D42S2
marker = D42S3
trait_marker = MOD
trait = Trait1
covariate = Cov, binary, affected=1, unaffected=2, missing=3
```
- 5. A name attribute should be specified for allele and may optionally be specified for marker parameters. It should be specified whenever the name of the field in the pedigree data file is not the same as the name that appears in the marker locus description file. If a name attribute is not specified, the marker name is assumed to be the field name. The order in which these fields are specified is arbitrary and not all the fields need appear in the data file.
- 6. For frequency adjustment, add attributes to the marker/allele parameter within the pedigree block. For example:

```
pedigree
{
   marker = D1S111, minimum_allele_freq = p
}
```
will replace with p all frequencies less than p , and then the frequencies will be normalized to sum to 1. The maximum_allele_freq parameter works in an analogous manner.

- 7. This will set all allele frequencies equal to 1/(number of alleles).
- 8. This will complement all allele frequencies and then normalize them to sum to 1. In other words, the frequencies listed in the locus description file will be individually complemented, and the complement added to a sum for all alleles at the locus. Each complemented frequency is normalized by dividing by the sum of the complemented frequencies.
- 9. A trait_marker parameter should be included for each trait in the data file that is to undergo a model-based linkage analysis. Thus the trait becomes like a marker and has requirements similar to those of a marker parameter, and hence is called a trait-marker. Instead of mixing markers and trait-markers in the same locus description file, each trait-marker should have an entry in the trait locus description file. THOSE NOT FOUND IN THE TRAIT LOCUS DESCRIPTION FILE WILL NOT BE ANALYZED.
- 10. The marker_list parameter specifies multiple markers simultaneously. This is prefered over specifying markers individually when markers all have the same formatting and there are many markers in the pedigree file. The marker_list parameter has the following attributes, corresponding to the same attributes as used for specifying markers individually:
	- x_linked
	- y_linked
	- missing
	- delimiter
	- minimum
	- maximum
	- equal
- complement
- 11. The start attribute indicates the first marker in the set, and the end attribute specifies the last. start and end are required attributes if the marker_list parameter is specified, and their omission prevents parsing the marker list (a fatal error is generated). Both the start marker and end marker must be present in the pedigree data file, with end following start. A fatal error will result if this is not the case. A marker_list with start and end being the same value will be interpreted as a single marker.
- 12. Multiple marker lists can be specified, but they may not overlap. Similarly they may not overlap with any other fields. When multiple marker lists and fields are not disjoint, a fatal error is issued.
- 13. For X-linkage, hemizygous males need to be coded as homozygotes. Y-linkage is implemented only in the MARKERINFO program.

3.2.6 The marker Block

The marker block sets global options on how to read and interpret marker genotype fields in the pedigree data file. Unless otherwise noted, all parameters and their corresponding attributes must be specified within a marker block of a parameter file which starts with a marker parameter.

The following table shows the syntax for a marker parameter:

The following table lists the parameters and attributes that may occur in a marker block.

Notes

- 1. See notes 6 through 8 of Section [3.2.5.4.](#page-49-0) Any value specified for parameters and attributes in this marker block will be used for all marker and allele fields in the pedigree data file unless each marker or allele statement in the pedigree block overrides them otherwise.
- 2. This option causes all marker data to be read in as covariate values without using a function block, with the result that any analysis that uses "marker" data (see [3.2.5.4\)](#page-49-0) cannot be performed. The value for covariate_function specifies the marker function: additive, dominant, or recessive. The base_allele attribute specifies the specific allele that will be considered to be 'Ai' in the marker function (see [3.2.7.3\)](#page-63-0). Note that this option allows a marker to be used for some programs (e.g. TDEX) but is not intended for X-linkage: for X-linkage male hemizygotes must be coded as homozygotes unless allow_hemizygote attribute is set to true, then marker data with only one allele specified, the other a missing value, will be treated as homozygote.

3.2.7 The function Block

User-defined function parameters specify the creation of new traits or covariates as a function of existing pedigree variables. Like other configuration parameters, function parameters may appear anywhere in the parameter file, but they are processed immediately after the pedigree data are read, IN THE ORDER IN WHICH THEY APPEAR. Thus, variables created by previous functions can be used in the specification of subsequent functions. Once created, a function variable may be used just like a trait or covariate read from a pedigree data file in all S.A.G.E programs that use traits and covariates in the analyses.

The following table shows the syntax for a function parameter:

Notes

1. If the list attribute is included, then the given function block will be executed for every item in the given list. The possible lists include all traits (traits and covariates, that is) or all markers given in the pedigree block. That is, the user may include either list=traits or list=markers. S.A.G.E. will interpret this to mean that it should generate a new, userdefined trait for every trait (or marker) in the list. To make the process more intuititve, S.A.G.E. will prepend the name of the newly-generated trait with the user-given name. Also, S.A.G.E. will recognize the \$name\$ token in both the function expressions and constant expressions; for each element in the list, S.A.G.E. will substitute the element's name for the \$name\$ token. For instance, consider a user who wants to apply the dominant() function to all markers present in the data. To apply the dominant() function to a single marker, the user would enter the following function block: function {trait = dom, expression $=$ "dominant (marker_name, 'A')"} (see [3.2.7.3\)](#page-63-0). If, however, the user wants to use the list attribute to apply the dominant() function to all markers, the user would enter the following function block: function, list = markers {trait = dom, expression = "dominant (marker_name, 'A')"}. Notice that the user has changed two aspects of the function block: the list = markers portion has been added, and the explicit name of the marker has been changed to the \$name\$ token. S.A.G.E. will now apply the function block for every marker specified in the pedigree block using marker, allele, and marker_list parameters. If, for instance, three markers M1, M2, and M3 are specified, then S.A.G.E. will create three new traits (dom M1, dom M2, and dom M3). Also, note that the $\text{\$name$toome$}$ token

can be used in both the function expression itself and in any of the user-defined constants. The following are all valid uses of the list attribute:

```
function, list = markers
{
   trait = dom, expression = "dominant ($name$, 'A')"
}
function, list = traits
{
   trait = squared, expression = "\text{\$name$ * } \text{\$name$"}"
}
function, list = traits
{
   constant = c, expression = (0.275)trait = added, expression = "\text{name} + c"
}
```


The following table lists the parameters and attributes that may occur in a function block.

Notes

1. The following constants^{[17](#page-60-0)} may be used for expression:

- 2. The two possible function variable types are trait and covariate. Covariate field is a generic designation and convey no suggestion of how the field is to be used. Trait fields are typically selected by many analyses to be used as major variates. Thus these parameters simply provide hints to make reasonable use of phenotypic information. Refer to the programspecific documentation for specific information on the behaviors of these parameters and how they may be overridden.
- 3. The value may be a character string representing the name of a new trait or covariate; however, THE FIRST CHARACTER MAY NOT BE A DIGIT. The name may not be that of an existing pedigree variable. Note that S.A.G.E. is CASE INSENSITIVE with respect to the names of traits and covariates. Thus the name "mean_bmi" is considered identical to "mean_BMI".
- 4. The value of expression should be an algebraic expression referring to one or more existing variables (traits, covariates or markers, either read from the pedigree data file or previously created as function variables) as well as allowable operators, elementary functions and constants. The variable name used for an expression may be specified by any character string, BUT THE FIRST CHARACTER OF THE STRING MAY NOT BE A DIGIT. Expressions should always be enclosed in double quotes (" "), and MUST BE ALL ON ONE LINE.

Examples to derive a new trait from existing traits:

¹⁷The two names *e* and *pi* are reserved and may not be used as the names of traits or covariates.

```
function
{
  # Create trait x from traits HDL and LDL
  trait = x, expression="log(HDL) - log(LDL)"
}
function
{
  # Create trait x from traits HDL and LDL
  trait = x, expression="log(HDL / LDL)"
}
```
The above two functions are equivalent. Note also that if LDL is 0, this trait is undefined (and hence a missing value is assigned to it).

- 5. Variable names are not case sensitive, but elementary function and constant names are.
- 6. A missing value for any of the variables in a function expression will result in a missing value for that function variable.
- 7. The time_limit parameter is provided to avoid situations where the calculation of values takes an inordinate amount of time. In most cases it need not be changed.

Example:

```
function
{
  # Creates, from variables h1 and h2, the covariate " average" whose
  # value is 1 if (h1 + h2)/2 is greater than .275, and 0 otherwise.
  # If the program cannot evaluate an expression in 2 seconds or less,
  # it will abort, giving a fatal error message.
 time_limit=2
 constant=gamma, expression = .275
  covariate = "average", expression = ((h1 + h2)/2 > gamma")}
```
The followng sections describe the operators and functions that may be used in function block expressions.

3.2.7.1 Operators

The following table indicates the operators that may be used. Those below the empty row are used to evaluate either zero (false) or one (true).

a If both operands are in integer form (contain no decimal point and are not in scientific notation), the result is integer also. Thus, $2.0 / 3 = .66667$, but $2 / 3 = 0$.

^{*b*}The use of the comparison operator (two equal signs), "==", creates a logical expression whose evaluation results in either 1 (if true) or 0 (if untrue).

Operators are evaluated in order of operator precedence from highest to lowest in the following list. Except when there are parentheses (see below), all operators of an equal precedence are evaluated before operators of lower precedence (from left to right, except for comparison operators which are evaluated from right to left). Operator precedence from highest to lowest is as follows (operators on

the same line have equal precedence):

Operator precedence may be overridden by parentheses. Expressions in parentheses are evaluated first (BRACKETS OR BRACES MAY NOT BE USED). Multiple parentheses are permissible; the computation starts within the innermost parentheses and works outwards. For example, we may have expression = " $(x + (y * z))$ ".

3.2.7.2 Elementary Functions

The following elementary functions may be used:

3.2.7.3 Marker Functions

The following functions are available for markers. In these functions the second argument (allele value) must be in single quotes as shown:

• dominant(marker, 'Ai') or dom(marker, 'Ai')

returns the value 1 or 0 based on the alleles present at the specified marker locus as follows:^{[18](#page-63-1)}

 A_i/A_i , A_i /*A** returns 1

*A**/*A** returns 0,

where A^* is any allele other than A_i .

¹⁸The examples assume that / is the allele delimiter within genotypes. However, a different delimiter could be used *.*

• recessive(marker, 'Ai') or rec(marker, 'Ai')

returns the value 1 or 0 based on the alleles present at the specified marker locus as follows:

 A_i/A_i /*Aⁱ* returns 1

 $A_i/A^*, A^*/A^*$ returns 0,

where A^* is any allele other than A_i .

• additive(marker, 'Ai') or add(marker, 'Ai')

returns the value 2, 1, or 0 based on the alleles present at the specified marker locus as follows:

 A_i/A_i /*Aⁱ* returns 2

 A_i/A^* /*A** returns 1

*A**/*A** returns 0,

where A^* is any allele other than A_i .

• genotype(marker, 'Ai', 'Aj') or gen(marker, 'Ai', 'Aj')

returns the value 1 or 0 based on the alleles present at the specified marker locus as follows:

 A_i/A_j , A_j /*Aⁱ* returns 1

Ai /*A**, *A^j* /*A**, *A**/*Aⁱ* , *A**/*A^j returns 0,*

where A^* is any allele other than A_i or A_j .

Here are some examples.

1. An ABO example:

```
function
{
   # Creates, from marker ABO, the covariate x whose value is 1 if marker
   # ABO genotypes are AB or BA, and 0 otherwise.
   covariate = x, expression = "dom(ABO, 'A') and dom(ABO, 'B')"
}
```
2. Equivalent ABO example:

```
function
{
   # Creates, from marker ABO, the covariate x whose value is 1 if marker
   # ABO genotypes are AB or BA, and 0 otherwise.
   covariate = x, expression = "gen(ABO, 'A', 'B')"
}
```
Note: If ABO is missing, the trait x will also be missing.

3. Another marker example:

```
function
{
   # Creates, from marker D42S8 and trait z, a covariate, y, whose value
   # is z if allele q1 is present at marker D42S8, and 0 otherwise.
   covariate = y, expression = "dominant (D42S8, 'q1') * z"}
```
3.2.7.4 Mean-Adjusted and Variance-Adjusted Data

S.A.G.E. provides the option of generating mean-adjusted, variance-adjusted or standardized values for each class of a stratification variable of a given trait or covariate. There are two basic steps to creating an adjusted variable:

- 1. Specify the classes of the stratification variable.
- 2. Define a new variable to be adjusted with respect to these classes.

The newly created variable can then be used in a S.A.G.E. analysis.

3.2.7.4.1 Specifying the Classes for Adjusting Data

Specify each class within a function block as the values of an expression attribute for the covariate parameter^{[19](#page-65-0)}. The following example shows how to create three classes of a covari-ate^{[20](#page-65-1)} named "Age":

```
function
{
   covariate = class1, expression = "(Age \leq 15)"
}
function
{
   covariate = class2, expression = "(Age > 15 and Age <= 30)"
}
function
{
   covariate = class3, expression = "(Age > 30)"
}
```
IMPORTANT! It is essential that the classification scheme partitions the data into exhaustive and mutually exclusive subsets with respect to the classification variable (" Age", in this case). If the data are not partitioned correctly, the resultant mean- and variance-adjusted variables will not be reliable.

3.2.7.4.2 Creating a Mean-Adjusted Variable

Once the classes of the stratification variable have been defined, the mean-adjusted values of some other trait (or covariate) can be calculated using the classes of the stratification variable that defines classes of the trait. Specify the mean-adjustment within a function block as the value of an expression attribute for the trait parameter^{[21](#page-65-2)}. Assuming the data file lists a trait or covariate called "BP", the following example shows how to create a new mean-adjusted variable named "BP_AgeAdjMean":

¹⁹The classes could also be created from a trait; however, it usually makes more sense to create them from a covariate.

 20 In this example, "Age" is assumed to be a covariate; however the stratification variable could also be a trait.

²¹The variable could also be created for a covariate.

```
function
{
   trait = BP_AgeAdjMean, expression = "mean_adj(BP, 10, class1, class2, class3)"
}
```
Note the use of the special keyword, mean_adj. This is what tells S.A.G.E. to add a new set of information to the internal computer representation of the pedigree file.

The value of the second argument to mean_adj (*10* in this example) determines the minimum number of items required for the classes. If, after the data have been stratified, any of the resultant classes has less than the minimum number of entries, then a special algorithm is employed to "borrow" values from neighboring classes in the ordered list of values until the minimum number has been reached for the underrepresented class. Note that, for the resulting mean-adjusted variable to be meaningful, the classes of the stratification variable must be in natural order.

The variable, *BP_AgeAdjMean*, essentially becomes a new trait, and is surreptitiously added to the internal representation of the data file (the original file is left unchanged). In this case, there are three different means computed:

- \bar{x}_1 : the mean BP for individuals whose age is less than or equal to 15,
- \bar{x}_2 : the mean BP for individuals whose age is greater than 15 and less than or equal to 30,
- \bar{x}_3 : the mean BP for individuals whose age is greater than 30.

If *BPⁱ* is the blood pressure value for individual *i* , then the value of " BP_AgeAdjMean" *for that individual* will be

- $BP_i \bar{x}_1$; if the individual's age is less than or equal to 15,
- $BP_i \bar{x}_2$; if the individual's age is greater than 15 and less than or equal to 30,
- $BP_i \bar{x}_3$; if the individual's age is greater than 30.

3.2.7.4.3 Creating a Variance-Adjusted Variable

The procedure for creating a variance-adjusted variable is analogous. The following example shows how to create a new variable named "BP_AgeAdjVar":

```
function
{
   trait = BP_AgeAdjVar, expression = "var_adj(BP, 10, class1, class2, class3)"
}
```
Here, the required keyword is var adj, and the resultant values of BP_AgeAdjVar (for an arbitrary individual *i*) will be:

- BP_i/s_1 : if the individual's age is less than or equal to 15,
- BP_i/s_2 : if the individual's age is greater than 15 and less than or equal to 30,
- *BP*^{*i*}/*s*₃: if the individual's age is greater than 30,

where s_i ($i = 1, 2, 3$) is the sample standard deviation of the trait *BP* for age class i .

3.2.7.4.4 Creating a Z-Score Variable

A standardized variable is obtained as follows:

```
function
{
   trait = BP_AgeZScore, expression = "z_score(BP, 10, class1, class2, class3)"
}
```
In this last example, the required keyword is z gcore, and the values of BP AgeZScore (for an arbitrary individual *i*) will be:

- $(BP_i \bar{x}_1)/s_1$: if the individual's age is less than or equal to 15,
- $(BP_i \bar{x}_2)/s_2$: if the individual's age is greater than 15 and less than or equal to 30,
- $(BP_i \bar{x}_3)/s_3$: if the individual's age is greater than 30.

3.2.7.4.5 Creating Adjusted Variable without Classes

It is also possible to create an adjusted variable that does not depend on the classes of a stratification variable. The result is simply the mean-adjusted, variance-adjusted or standardized value of a given variable with respect to the entire sample.

To create a mean-adjusted variable (*BP_AdjMean*) from the variable *BP* , write:

```
function
{
   trait = BP\_AdjMean, expression = "mean_adj(BP)"
}
```
To create a variance-adjusted variable (*BP_AdjVar*) from the variable *BP* , write:

```
function
{
   trait = BP\_AdjVar, expression = "var_adj(BP)"
}
```
To create a standardized variable (*BP_Normalized*) from the variable *BP* , write:

```
function
{
   trait = BP_Normalized, expression = "z_score(BP)"
}
```
3.2.7.5 Data Trimming and Winsorizing

S.A.G.E. provides a way to minimize the adverse impact of outlier data by creating variables that either trim or Winsorize the tails of the distributions as shown in the following example:

After trimming:

Data that are subjected to the trim function are effectively thrown out of the analysis, whereas Winsorized data are revalued to a quantity that corresponds to some critical point along the distribution.

3.2.7.5.1 Creating a Trimmed Variable

Create a trimmed variable using the trim S.A.G.E. keyword as in the following example:

```
function
{
  trait=LNIGE_trim, expression= "trim(LNIGE,0.02)"
}
```
The trim function takes two arguments:

- 1. the name of a trait or covariate (here " LNIGE") previously specified in the pedigree block
- 2. a value $\gamma \in (0, 1)$, representing the "amount" of data to be trimmed (the value 0.02 will result in trimming 1% of the values in each tail of the distribution).

The newly created variable, *LNIGE_trim* , can be used in the same manner as any other trait or covariate within S.A.G.E. applications.

3.2.7.5.2 Creating a Winsorized Variable

Create a winsorized variable using the winsor S.A.G.E. keyword as in the following example:

```
function
{
  trait=LNIGE_wins, expression = "winsor(LNIGE,0.02)"
}
```
The winsor function takes two arguments:

- 1. the name of a trait or covariate (here " LNIGE") previously specified in the pedigree block
- 2. a value $\gamma \in (0, 1)$, representing the "amount" of data to be winsorized (the value 0.02 will result in 1% of the values in each tail of the distribution being replaced by the corresponding 1 and 99 percentiles).

The newly created indicator variable, *LNIGE* _ *wins*, can be used in the same manner as any other trait or covariate within S.A.G.E. applications.

3.2.7.6 The Transmitted and Untransmitted Allele Indicators (TAI and UTAI)

The problem of performing a transmission disequilibrium test (TDT) to assess the linkage between a marker locus and a quantitative trait was addressed in a paper by George et al (1999), who proposed a linear-regression approach in which the trait (assumed to be quantitative) is the dependent variable, *Y* . The primary independent predictor variable in the model, *X* , is an indicator variable that reflects whether or not a given allele was transmitted to the individual from a heterozygous parent (see Figure 1). The authors refer to X as a *transmission status variable* which is referred to here by the slightly more accurate term: *transmitted allele indicator* (TAI).

Figure 1: Offspring who are informative for linkage, from relevant parental matings. *A* is the associated allele of interest, and *X* is the transmitted allele indicator variable such that *X* =1 if *A* was transmitted from a heterozygous parent, and *X* =0 otherwise.

For example, consider a diallelic locus $\{A, a\}$ and suppose we wish to determine the TAI with respect to allele 'A'. Then the TAI values computed for a given individual would be as shown in the table below, which also indicates the UTAI as well^{[22](#page-69-0)}:

²²When the marker locus has more than two alleles, we appropriately extend this indicator to make use of the maximum amount of information available in an unbiased fashion. See the theory section of the TDTEX program in this manual.

To specify TAI and/or UTAI variables for a single marker, create a function block that defines the new variables using the tai and utai keywords as in the following example:

```
pedigree
{
   .
   .
   .
   allele = "M1A", name = "M1" #marker specified in pedigree block
   allele = "M1a", name = "M1" #marker specified in pedigree block
}
function
{
   trait = M1A\_tai, expression = "tai(M1, A)"
}
function
{
   trait = M1a_tai, expression = "tail(M1, a)"}
function
{
   trait = M1A_utai, expression = "utai(M1, A)"
}
function
{
   trait = M1a_utai, expression = "utai(M1, a)"
}
```
The newly created indicator variables, *M1A_tai, M1a_tai* , *M1A_utai* and *M1a_utai* , can be used in the same manner as any other trait or covariate within S.A.G.E. applications.

3.3 Locus Description Files

The marker locus description file and the trait locus description file follow the same format as each other and contain records that define allele frequencies and phenotype to genotype mappings. The marker locus description file contains a record for each marker locus. The trait locus description file contains a record for each trait, or "trait-marker", that is to undergo a model-based linkage analysis. A record must be included in the corresponding locus description file for each marker locus or traitmarker to be analyzed, and these records may appear in any order. All marker loci and trait-markers listed in the parameter file and/or the genome description file should be present in the corresponding locus description file: THOSE NOT PRESENT THERE ARE IGNORED. In the case of fully penetrant and codominant markers, the program FREQ can be used to prepare the marker locus description file.

The locus description file should contain the following items for **each** locus to be analyzed:

- 1. The name of the locus.
- 2. A set of records that give the allele frequencies. The records should follow this format:

allele_symbol = *population allele frequency*

The user should supply the information for the items on both sides of the "=" symbol. There can be any number of spaces before or after the equal sign as long as the allele symbol and its frequency remain on the same line. There should be only one allele symbol per line. The allele symbol can consist of up to 10 characters. Allele frequencies should sum to 1. Otherwise, they will be normalized to do so. It is also permissible to list just the alleles, leaving out " = allele frequency" from every line; when this is done, equal allele frequencies are substituted for each allele listed for that locus. This option is useful when the marker locus description file is used in conjunction with a program that does not use allele frequencies (e.g., ASSOC).

- 3. A semicolon indicating the end of the alleles. This semicolon can be either on a line by itself or on the same line following the last population allele frequency of the set.
- 4. A set of records that defines the *phenosets* (i.e., the sets of genotypes compatible with each marker phenotype). The records should follow this format:

$$
phenotype_symbol = \{AI_1/A2_1[=P_1],...,AI_m/A2_m[=P_m]\}
$$

where

. . .

 $A1₁$ is the symbol for allele #1 in the first genotype of this phenoset;

 $A2₁$ is the symbol for allele #2 in the first genotype of this phenoset;

 $A1_m$ is the symbol for allele #1 in the m-th (last) genotype of this phenoset;

 $A2_m$ is the symbol for allele #2 in the m-th (last) genotype of this phenoset,

and $P_1...P_m$ are the penetrance values of the phenotype, i.e., the probabilities of the phenotype given the genotype. A phenoset is not required for a codominant locus that is fully penetrant,
and the penetrance values are strictly optional. If no value is indicated, the phenotype is assumed to be fully penetrant and a value of 1 is assumed.

There can be any number of spaces before or after the equal sign(s). The phenoset should begin with either a left curly brace ({) or less-than symbol (<), and end with a corresponding right curly brace (}) or greater-than symbol (>). The first and second allele of each genotype must be separated by a slash (/) or otherwise specified allele delimiter. Consecutive genotypes within the phenoset should be separated by a comma. This record may wrap onto as many lines as necessary. Complete the set by repeating this record for each phenotype at this locus. Any phenotype symbol that is not included here is interpreted as a missing phenotype value. The order of the alleles in a genotype has no effect.

In the following example, A, B, C, A1, A2, and O are the values of the alleles and, 1, 2, 3, 4, 5, and 6 are the values of the marker phenotypes as entered in the pedigree data file.

```
LOCA # locus name
A = 0.5 #(allele/phenotype names are arbitrary
B = 0.25 \# and need not be in any particular order)
C = 0.25# first semicolon means no more alleles to list
1 = \{A/A, A/B, A/C\} #(A is dominant over B and C, and
2 = {B/B, B/C} #B is dominant over C)
3 = \{C/C\}; # second semicolon means no more phenotyes/phenosets
ABO # name of next locus<br>A1 = 0.1904 # allele frequencies
                       # allele frequencies should sum to 1.0 (otherwise
A2 = 0.0612 # they will be normalized to do so.)
B = 0.07280 = 0.6756;
1 = \{ A1/A1, A1/A2, A1/O \} #(A1 is dominant over A2 and 0)
2 = \{ A1/B \}3 = \{ A2/A2, A2/O \} #(A2 is dominant over 0)
4 = \{ A2/B \}5 = {B/B, B/O} #(B \text{ is dominant over } 0)6 = \{ 0/0 \};
```
If a locus is fully penetrant and codominant, it is not necessary to include the records for phenotypes. The program will generate the phenotype symbol by concatenating the two allele symbols of the genotype and putting a delimiter character between them (typically a /, but this can be modified in the parameter file). However, the semicolon indicating the end of the phenotypes still has to be included.

For example, the following two locus descriptions are equivalent:

Trait-markers are specified similarly. As an example, suppose we have a trait "Disease", and an underlying model with two disease alleles (allele 1 has frequency 10% and allele 2 has frequency 90%) and two phenotypes ($A =$ affected, $U =$ unaffected). Suppose that we are assuming that allele 1 predisposes toward the expression of affection, and furthermore that it is recessive to allele 2.

Our penetrance table might look something like this:

i.e., 60% penetrance and a sporadic rate of 1%. The trait locus description file would then contain the following entry:

Disease $1 = 0.10$ $2 = 0.90$; $A = \{ 1/1 = 0.6, 1/2 = 0.01, 2/2 = 0.01 \}$ $U = \{ 1/1 = 0.4, 1/2 = 0.99, 2/2 = 0.99 \}$;

Note that the trait need not be binary (any number of phenotypes may be specified), and the locus may have more than two alleles. For any particular genotype, the sum of all (here two) penetrances should equal 1.

3.4 Genome Description File

The genome description file describes the genomic region(s) used in analyses that require the order of, and distances between, linked marker loci. A genome is defined with at least one genomic region. This region contains the names of sequentially ordered marker loci and the distances or recombination fractions between pairs of adjacent markers. A map function is used to translate genetic map distances to and from recombination fractions. The general form of the file is as follows:

```
genome = "genome name"[,map="map function"]
{
   [region1]
   [region2]
   [region3]
   .
   .
   .
}
```
The genome name can be any name desired. The map attribute allows specification of a map function, which can be either the Haldane or Kosambi map functions. If no map function is supplied, Haldane is assumed. The map function is used soley to convert genetic distances between consecutive markers into recombination fractions. As is the case for all other linkage programs, S.A.G.E. does not incorporate interference into linkage analysis. Map functions are not used during singlemarker (two-point) analysis.

Each genomic region is described as follows:

```
region="region name"
{
   [marker and distance parameters]
}
```
The region name is used to identify the region being defined. If no name is specified, "region n" is used, where n is the number of the region within the genome. The attribute x_linked is needed after the region name to indicate the region to be X-linked as follows:

```
region="region name", x_linked
{
   [marker and distance parameters]
}
```


The following parameters are available within a region sub-block:

Notes

- 1. In the program output, the first marker in each region is located at an absolute distance of 0.0 cM and all further markers are measured from this location in the map units specified by the map attribute. It is therefore advisable to include an initial marker "pter" as the initial marker for each chromosome.
- 2. There is a maximum of one genetic distance or recombination fraction value between each pair of markers. When doing multi-point analysis, there must be either a genetic distance or recombination fraction value between each pair of adjacent markers.
- 3. The S.A.G.E. GUI provides a genome map file wizard that can convert the marker coordinates to the genetic distance values between each pair of markers. Please refer to the GUI manual.

Here is an example of a typical Genome Description file:

```
genome
{
  # No genome name or map function specified.
  # Haldane map function is assumed
  # No region name specified, so the name is
  # assumed to be "region 1"
  region
  {
     marker = " pter" # Dummy marker name for p-terminal end of the chromosome
     distance = 154.7100 # Initial distance is measured from pter
     marker = "D4S2999" # at 154.7100 cM
```


}

. }

3.5 IBD Sharing File

The IBD sharing file, produced by the S.A.G.E. program GENIBD, stores the probability distribution of allele-sharing identical-by-descent (IBD) between pairs of individuals at specific locations. The header of the file contains the n names $(L1, L2, Ln)$ of the locations at which IBD sharing information is stored for each pair of relatives. These locations are referred to as markers, even though they may not correspond to observed marker loci in a given dataset. The body of the file contains a line for each pair of individuals that includes the following fields:

- pedigree ID
- First individual ID
- Second individual ID
- f_0 : The probability that the pair shares 0 alleles IBD at marker L_1
- f_{1m-1p} : The probability that the pair shares 1 maternal allele minus the probability that it shares 1 paternal allele IBD at marker L_1
- f_2 : The probability that the pair shares 2 alleles IBD at marker L_1

. . .

- f_0 : The probability that the pair shares 0 alleles IBD at marker L_n
- f1*m*−1*p*: The probability that the pair shares 1 maternal allele minus the probability that it shares 1 paternal allele IBD at marker L_n
- f_2 : The probability that the pair shares 2 alleles IBD at marker L_n

The probability that a pair shares one allele IBD at a given marker is $f_1 = 1 - f_0 - f_2$, where f_0 and f_2 are the probabilities that the given pair shares 0 and 2 alleles IBD at the marker. Similarly, the estimated proportion of alleles shared IBD is $f_2 + \frac{1}{2}$ $\frac{1}{2}f_1$. These probabilities are conditional on the pedigree and marker information available and are usually denoted \hat{f} in the literature.

Notes

- 1. IBD sharing files are generated in a prior analysis by the program GENIBD and used as input to the programs, LODPAL, SIBPAL, and RELPAL.
- 2. Packages other than S.A.G.E. may be able to use IBD sharing files produced by GENIBD as input, but the format in S.A.G.E. is subject to change.
- 3. The number of markers may be very large, so each line of the IBD sharing file can be extremely long. Loading these files into text-editors, especially those that wrap or truncate long lines, is not recommended.
- 4. IBD sharing files may be extremely large if there are many pairs and markers. When performing analyses on extremely large pedigrees and/or genome screens, IBD sharing files may consume disk space in excess of a gigabyte. Thankfully, IBD sharing files are amenable to many forms of data compression when not in use.

3.6 Information Output Files

An information output file is generated by all S.A.G.E. programs and contains diagnostic output generated during program execution. Typically, this includes information about how pedigree data files were read and diagnostic information on pedigree structure, traits, covariates and marker loci. This file is named "*program.inf*", indicating the name of the specific program that was run^{23} run^{23} run^{23} . AL-THOUGH NO ANALYSIS RESULTS ARE STORED IN THIS FILE, THE USER SHOULD MAKE A HABIT OF EXAMINING THE CONTENTS OF THIS FILE BEFORE OPENING ANY OTHER FILE PRODUCED BY A PROGRAM.

All S.A.G.E. programs that read trait or marker locus description files or genome description files generate the genome information File. This file contains diagnostic information on each marker or trait and genotype. This file is named "*genome.inf* ". Although no analysis results are stored in this file, errors relating to the markers and traits may be there.

3.7 Analysis Output Files

All S.A.G.E. programs produce one or more analysis output files, which contain the results of the analyses. The number of analysis output files, their names and contents are program specific. Analysis output files may even correspond to other S.A.G.E. input file types. E.g., the analysis output file from GENIBD is an IBD sharing file that is an input file for SIBPAL.

 23 Eg., fcor.inf, mlod.inf, segreg.inf, etc.

Chapter 4

AGEON

This program fits by maximum likelihood a mixed distribution for a binary trait (affected versus unaffected) with variable age of onset. The parameters estimated are the susceptibility to disease at age infinity (γ), the mean (μ) and variance (σ^2) of a power-normal age of onset distribution, and the power parameter (λ_1) ; a shift parameter (λ_2) may be specified. The mean, variance, and logit of susceptibility can each depend linearly on covariates. A class susceptibility covariate with six class values is generated according to the values of a parental binary trait. The parameter estimates can be used in a special function to produce of a set of eight new variables, for a binary trait with variable age of onset, any one of which may be used in a SIBPAL analysis.

4.1 Limitations

No account is taken of ascertainment or familial correlations; i.e. all individuals are assumed to be randomly sampled. This does not affect the validity or robustness of any SIBPAL analysis. Genetic susceptibilities are not estimated for classes with fewer than 5 informative members. If for any reason the power parameter λ_1 is fixed by the user at 0, then the value of the shift parameter λ_2 *must* be set such that λ_2 + the minimum value for age-of-onset or age-at-exam, whichever is smaller, $is > 0$.

4.2 Theory

4.2.1 Basic notation

Let the number of sibs in the sample be *n* .

Let *i* index the sib: $i = 1, 2, \ldots, n$.

Let *j* index the class of sib generated according to the values of a parental binary trait.

Let a_i denote age of onset and a'_i the age at examination, the latter being available for all unaffected persons to be included in the analysis.

Define φ and Φ be the normal density and cumulative distribution functions, and *h* be the transformation function as follows:

$$
\varphi(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\},
$$

$$
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left\{-\frac{1}{2}u^2\right\} du, \text{ and}
$$

$$
h(x) = \begin{cases} \frac{(x+\lambda_2)^{\lambda_1}-1}{\lambda_1}, & \text{if } \lambda_1 \neq 0 \\ ln(x+\lambda_2), & \text{if } \lambda_1 = 0 \end{cases}.
$$

AGEON uses an extension of the Box and Cox (1964) transformation to estimate λ_1 , μ_i , σ_i^2 and γ_i , wherein γ_i is automatically a function of a parental binary trait (γ_j), assumed here to be affection status, and other user-specified covariates, and the last three parameters are defined possibly as functions of user-specified covariates $(x_1, x_2, ...)$, as follows:

$$
\mu_i=\mu+\xi_1x_{1i}+\xi_2x_{2i}+\cdots,
$$

$$
\sigma_i^2 = \sigma^2 + \xi_1 x_{1i} + \xi_2 x_{2i} + \cdots
$$
, and

$$
\gamma_i = \frac{e^{\theta_i}}{1 + e^{\theta_i}}, \text{ where } \theta_i = \gamma_j + \xi_1 x_{1i} + \xi_2 x_{2i} + \cdots.
$$

AGEON allows the susceptibility and the mean and variance of the power (λ_1) transformed shifted (λ_2) age of onset to be dependent on different sets of covariates, and in each case the maximum likelihood estimates of the regression coefficients are obtained.

As done elsewhere in S.A.G.E., the transformation is applied to both sides of the equation for the mean age of onset, so the estimates $\hat{\mu}$, $\hat{\xi}_1$, $\hat{\xi}_2$, ... are asymptotically median unbiased estimates on the original scale, rather than on the transformed scale, and so more meaningful. But note that the estimate of variance is now on the transformed scale, and is not very relevant unless $\lambda_1 = 1$, in which case the variance is not changed.

4.2.2 Classification

AGEON classifies sibs according to whether or not each parent's affection status is known and, if known, whether each parent is or is not affected. Using '?' for unknown affection status, 'A' for affected, and 'U' for unaffected, an individual's class can be expressed as an order-independent combination of any two of those symbols according to the value of the parental affection status, as shown in the following table:

AGEON first estimates one overall susceptibility intercept, so that the class susceptibilites are equal, and then intercepts for each of the six classes, or for fewer classes if some of the classes are pooled; the corresponding likelihoods are used to perform an (asymptotic) likelihood ratio test that all the class susceptibilities are equal.

4.2.3 Likelihood

The log likelihood maximized is $\sum_{n=1}^{\infty}$ ∑ *i* $ln L(i)$, where $L(i)$ is the likelihood for the *i*-th sib. The likelihood $L(i)$ is given by:

As mentioned above, the mean μ_i , variance σ_i^2 , and susceptibility γ_i , may depend on covariates. In the case of the last, susceptibility, the logit is assumed to be a linear function of the covariates.

Because $a_i + \lambda_2$ must be positive to apply the Box and Cox (1964) transformation, prior to transformation $a_i + \lambda_2$ cannot strictly follow a normal distribution. This is usually of no consequence but, letting

$$
sign(x) = \begin{cases} 1, & \text{if } x \ge 0 \\ -1, & \text{if } x < 0 \end{cases}
$$

the maximization can also be performed using the following likelihoods, which allow for the truncation (see Pericak-Vance et al, 1983):

4.2.4 New Variables

The primary purpose of the AGEON program is to estimate the parameters needed to calculate either of two new quantitative variables that can be used in SIBPAL. These variables are to detect linkage to

- 1. genes that affect susceptibility to disease, and
- 2. genes that affect age of onset of disease.

The first quantitative variable, susceptibility to disease conditional on whether the individual *i* is affected or not by age a'_{i} , called the *susceptibility trait* (Schnell et al., 2012), is given by

$$
y_i = \left\{ \begin{array}{ll} 1 & , \text{if affected} \\ & \frac{\gamma_i - \gamma_i \Phi\left[\frac{h\left(a_i^{\prime}\right) - h\left(\mu_i\right)}{\sigma_i}\right]}{1 - \gamma_i \Phi\left[\frac{h\left(a_i^{\prime}\right) - h\left(\mu_i\right)}{\sigma_i}\right]} & , \text{if not affected by age } a_i^{\prime} \end{array} \right.
$$

where Φ is the standard cumulative normal distribution function, λ_1 and λ_2 are the transformation parameters, and γ is the susceptability.

The second quantitative variable, the disease age of onset is the *survival analysis residual* given by

$$
y_i = \begin{cases} 1 - \gamma_i \Phi\left[\frac{h(a_i) - h(\mu_i)}{\sigma_i}\right] & \text{, if affected at age } a_i \\ - \gamma_i \Phi\left[\frac{h(a'_i) - h(\mu_i)}{\sigma_i}\right] & \text{, if not affected by age } a'_i \end{cases}
$$

where again Φ is the standard cumulative normal distribution function, λ_1 and λ_2 are the transformation parameters, and γ is the susceptability.

Using one of these values of y as a quantitative trait can be more powerful in the usual Haseman-Elston test for linkage than using disease status as a simple binary trait (Zhu et al., 1997; Hanson and Knowler, 1998).

4.3 Program Input

4.3.1 Running ageon

A typical run of the AGEON program may use flags to identify the file types like the following:

>ageon -p data.par -d data.ped

or, rely on a set file order like the following:

>ageon data.par data.ped

where data.par is the name of the parameter file and data.ped is the name of the pedigree data file.

4.3.2 The ageon Block

An ageon block in the parameter file sets the options on how to perform an analysis using AGEON. The following table shows the syntax for an ageon parameter which starts the ageon block.

The following table lists the parameters and attributes that may occur in an ageon block.

Notes:

- 1. It is permissible for the age_onset and age_exam parameters to specify the same quantitative trait, in which case the value of this trait is assumed to be age of onset for affected persons and age at exam for unaffected persons. This can only be done if the age given for an affected person is the age of onset or unknown, i.e. this disallows the possibility of using the information on age at examination of an affected person when age of onset is unknown.
- 2. If the value of false is specified and any single individual's covariate value is missing, then that individual will be treated as uninformative for the purpose of the analysis. If true is specified, missing covariate values will be replaced with the covariate's mean value as calculated from the sample used in the analysis.
- 3. See section [4.2](#page-79-0) for details on the transformation theory implemented in this program.
- 4. The value of pool should be one or more algebraic expressions, where each expression refers to two or more default class names (??, ?A, ?U, AA, AU and UU) and equal (=) signs. Expressions should always be enclosed in double quotes (" "), and MUST BE ALL ON ONE LINE. Examples:

pool = "??=UU" # Pools the ?? class with the UU class. $pool = ''??=?A=AA"$ # Pools the ??, ?A and AA classes. $pool = "??=UU, ?A=AU" # Pools ?? with UU, and ?A with AU.$

4.3.2.1 The mean_cov Sub-Block

The following table lists the parameters and attributes that may occur in a mean_cov sub-block.

Notes:

1. The default is to include no mean covariates in the analysis. The means indicated in the mean_cov sub-block are a linear function of this covariate. All covariates are centered, the centering (average) value being included as part of the output.

4.3.2.2 The var_cov Sub-Block

The following table lists the parameters and attributes that may occur in a var_cov sub-block.

Notes

1. The default is to include no covariates in the analysis. The variances indicated in the var_cov sub-block are a linear function of this covariate . All covariates are centered, the centering (average) value being included as part of the output.

Examples:

```
ageon
{
   title = "analysis"
   affectedness = aff
   age_of_onset = ao
   age_of_exam = ae
   var_cov
   {
      covariate = cov2
   }
}
```
4.3.2.3 The suscept_cov Sub-Block

The following table lists the parameters and attributes that may occur in a suscept_cov sub-block.

Notes

1. The default is to include no susceptibility covariates in the analysis. The suscept_cov sub-block indicates which covariates are to modify the logits of susceptibilities. All covariates are centered, the centering (average) value being included as part of the output.

Examples:

```
ageon
{
  title = "analysis"
  affectedness = aff
   age_of_onset = ao
   age_of_exam = ae
```

```
suscept_cov
   {
      covariate = cov1
   }
}
```
4.3.2.4 The transformation Sub-Block

The following table lists the parameters and attributes that may occur in a transformation subblock.

Notes

1. If this parameter is fixed, it must be > 0 .

4.4 Program Output

AGEON produces several output files that contain results and diagnostic information:

4.4.1 Summary Output File

The summary output file contains descriptive information about each of the six classifications, and results for the case without truncation: final estimates, standard errors, and p-values of the parameters estimated in the model, including

- susceptibility intercept(s) and covariates
- mean intercept and covariates
- variance intercept and covariates
- transformation parameters (λ_1 and λ_2).

The file also includes a likelihood ratio test statistic (under the model assumptions) for the comparison of separate susceptibilities for each category of the classification variable and all the susceptibilities constrained to be equal.

Example:

Number of individuals in dataset 787

```
Number of analyzable individuals 787
  Number of analyzable invalid individuals 149
  Number of analyzable valid individuals 638
===========================
     MODEL DESCRIPTION
===========================
 Title AGEON Analysis 1
  Affectedness trait AFF
 Age-of-onset trait AO
  Age-at-exam trait AE
     ===============================
   CLASSIFICATION SYSTEM
===============================
  Using default classification system:
  ?? Both parents are unknown.
  ?A One of the parents is unknown, the other is affected.
  ?U One of the parents is unknown, the other is unaffected.
  AA Both parents are affected.
  AU One of the parents is affected, the other is unaffected.
  UU Both parents are unaffected.
==========================
   CLASS STATISTICS
==========================
==================
   CLASS ??
==================
 TOTAL NUMBER OF INDIVIDUALS USED IN ANALYSIS 60
  NUMBER OF INDIVIDUALS WITH AN AGE OF ONSET 25<br>MEAN OF AGE OF ONSET 25
  MEAN OF AGE OF ONSET
  VARIANCE OF AGE OF ONSET<br>NIIMBER OF INDIVIDIIALS AFFECTED
  NUMBER OF INDIVIDUALS AFFECTED<br>PROPORTION OF INDIVIDUALS AFFECTED 0.150000
 PROPORTION OF INDIVIDUALS AFFECTED 0.150000<br>MEAN OF AGE AT EXAM OF THE UNAFFECTED 74.816667
  MEAN OF AGE AT EXAM OF THE UNAFFECTED 74.816667<br>VARIANCE OF AGE AT EXAM OF THE UNAFFECTED 6.949722
  VARIANCE OF AGE AT EXAM OF THE UNAFFECTED
.
.
.
==================
    CLASS UU
==================
  TOTAL NUMBER OF INDIVIDUALS USED IN ANALYSIS 43
  NUMBER OF INDIVIDUALS WITH AN AGE OF ONSET 15<br>MEAN OF AGE OF ONSET 15
 MEAN OF AGE OF ONSET
  VARIANCE OF AGE OF ONSET 4.595556
  NUMBER OF INDIVIDUALS AFFECTED<br>PROPORTION OF INDIVIDUALS AFFECTED 0.209302
  PROPORTION OF INDIVIDUALS AFFECTED 0.209302<br>MEAN OF AGE AT EXAM OF THE UNAFFECTED 75.744186
 MEAN OF AGE AT EXAM OF THE UNAFFECTED 75.744186<br>VARTANCE OF AGE AT EXAM OF THE UNAFFECTED 5.446187
  VARIANCE OF AGE AT EXAM OF THE UNAFFECTED
====================================================================
    MAXIMIZATION RESULTS susceptibilities equal, no truncation
====================================================================
```


H0 ln likelihood susceptibilities free, no truncation -382.959654 H1 ln likelihood susceptibilities equal, no truncation -383.779624

4.4.2 Detailed Output File

The detailed output file contains all information present in the summary output file, and has the following additional information:

- first partial derivatives of the log likelihood for all parameters
- estimates for all four models (with/without truncation)
- variance-covariance matrices for all four models
- additional likelihood ratio test statistics for the models not listed in the summary output file.

Example of additional part of an AGEON detailed output file:

.

Mean covariates cov1 0.346353 0.172817 0.045054 -0.0000000337 Variance intercept 6.358667 1.578147 5.60e-05 0.0000000000 Variance covariates cov2 1.907274 1.214693 0.116376 0.0000000707 Transformation Lambda1 1.000000 Fixed Lambda2 0.050000 Fixed -- Final ln likelihood: -382.959654 =============================== Likelihood Ratio Test =============================== . .
=============================== Likelihood Ratio Test =============================== H0 ln likelihood susceptibilities free, using truncation -382.959654 H1 ln likelihood susceptibilities equal, using truncation -383.779624 2 * |H0 - H1| 1.639942 Degrees of freedom 5

P-value 0.896377
 $P-value$ 0.896377 == VARIANCE-COVARIANCE MATRIX susceptibilities equal, no truncation == -- Mean intercept Variance intercept cov1 cov2 ?? ?A ?U AA AU UU

-- Mean intercept 0.033182 0.057390 -0.003310 0.032535 0.010835 0.010835 0.010835 0.010835 0.010835 0.010835 Variance intercept 0.057390 2.439526 -7.59e-05 1.257216 0.041093 0.041093 0.041093 0.041093 0.041093 0.041093 cov1 -0.003310 -7.59e-05 0.029612 0.006981 -0.000758 -0.000758 -0.000758 -0.000758 -0.000758 -0.000758 cov2 0.032535 1.257216 0.006981 1.443488 0.021532 0.021532 0.021532 0.021532 0.021532 0.021532 ?? 0.010835 0.041093 -0.000758 0.021532 0.026250 0.026250 0.026250 0.026250 0.026250 0.026250 ?A 0.010835 0.041093 -0.000758 0.021532 0.026250 0.026250 0.026250 0.026250 0.026250 0.026250 ?U 0.010835 0.041093 -0.000758 0.021532 0.026250 0.026250 0.026250 0.026250 0.026250 0.026250 AA 0.010835 0.041093 -0.000758 0.021532 0.026250 0.026250 0.026250 0.026250 0.026250 0.026250 AU 0.010835 0.041093 -0.000758 0.021532 0.026250 0.026250 0.026250 0.026250 0.026250 0.026250 UU 0.010835 0.041093 -0.000758 0.021532 0.026250 0.026250 0.026250 0.026250 0.026250
AU 0.010835 0.041093 -0.000758 0.021532 0.026250 0.026250 0.026250 0.026250 0.026250 0.026250
UU 0.010835 0.041093 -0.000758 0.021532 0.0

=== VARIANCE-COVARIANCE MATRIX susceptibilities free, no truncation

===

Error: Matrix is not available.

=== VARIANCE-COVARIANCE MATRIX susceptibilities equal, using truncation ===

==

VARIANCE-COVARIANCE MATRIX susceptibilities free, using truncation ==

Error: Matrix is not available.

4.4.3 Pedigree and Parameter Output Files

For each of the four models maximized in an AGEON analysis, the two variables (susceptability trait and survival analysis residual) are calculated (for each individual) on the basis of each model's final estimates and are made available for subsequent analysis in an AGEON-generated pedigree file. This file, named for the analysis conducted (such as 'ageon_analysis*n*.ped') is created automatically; the user need not worry about explicitly requesting AGEON to generate the file. This pedigree file contains, for each individual, eight columns with the following names:

- 1. nt_equal_trait Susceptibility trait for the model with no truncation and susceptibilities equal.
- 2. nt_equal_residual Survival analysis residual for the model with no truncation and susceptibilities equal.
- 3. nt_free_trait Susceptibility trait for the model with no truncation and susceptibilities free or as pooled.
- 4. nt_free_residual Survival analysis residual for the model with no truncation and susceptibilities free or as pooled.
- 5. t_equal_trait Susceptibility trait for the model with truncation and susceptibilities equal.
- 6. t_equal_residual Survival analysis residual for the model with truncation and susceptibilities equal.
- 7. t_free_trait Susceptibility trait for the model with truncation and susceptibilities free or as pooled.
- 8. t_free_residual Survival analysis residual for the model with truncation and susceptibilities free or as pooled.

In addition to this pedigree file, the parameter file is also created, named for the analysis conducted (such as 'ageon_analysis*n*.par'), containing the corresponding pedigree block that describes the new pedigree file. Using these new files, you can now conduct analyses on the susceptibility traits and survival analysis residuals from any of four AGEON models.

Chapter 5

ASSOC

ASSOC assesses the association between a quantitative or binary trait and one or more covariates (which may include marker phenotypes that have been transformed into quantitative covariates) from extended pedigree data in the presence of familial correlations, simultaneously estimating familial variance components (and hence familial correlations and heritability). Given data on one or more independent pedigrees sampled at random, this program estimates (by maximum likelihood, assuming a generalization of multivariate normality) the parameters of a baseline model, as well as those of alternate models that include specified sets of covariates, and performs a likelihood ratio test for the significance of covariates not included in the baseline model. It also calculates numerically the standard errors of the estimates of all individual parameters in the model and performs an appropriate Wald test on each. In addition, ASSOC can perform tests that are robust to population stratification (e.g. QTDT) by use of a transmitted allele indicator (see [3.2.7.6\)](#page-69-0) and, using the GUI, independent model residuals produced by ASSOC can be easily imported into the program GMDR to perform a multidimensional reduction analysis with family data.

5.1 Limitations

Pedigrees must not have loops, and a string of mates of length more than three is not allowed; i.e. a person may have multiple mates, but none of the mates may have another mate (e.g: woman-manwoman-man is not allowed). If the sample size is small relative to the number of parameters being estimated, the likelihood may have multiple maxima. There is no guarantee that in such a situation the maximum found and reported by the program is the global maximum. Also, situations can occur in which it is not numerically possible to calculate the variance-covariance matrix of the estimates.

5.2 Theory

5.2.1 Description of the Model

To incorporate familial correlations and arbitrary covariates into a likelihood, we assume the correlation structure described in Elston, George and Severtson (1992) and the regression model similar to that described in George and Elston (1987). For individual *i* , let:

- Y_i = a quantitative trait
- x_i = a vector of covariates
- G_i = a random additive polygenic effect
- F_i = a random nuclear family effect
- M_i = a random marital effect
- S_i = a random sibship effect
- E_i = a random individual (environmental and/or measurement error effect)

 F_i is an effect common to all members of the same nuclear family; M_i is an effect that spouses share with each other; S_i is an effect that full sibs share with each other (and hence allows for dominance variance and common sibling environmental variance); and *E* is a person-specific random effect. Note that an individual may belong to several different nuclear families: together with a spouse and children, and/or together with sibs and parents; if a person has children by *k* different spouses, that person will belong to those *k* different nuclear families as a parent, and could additionally belong to a family as an offspring with sibs and parents. In these situations the person will have more than one distinct family effect.

Then the default model for a quantitative trait is of the form

$$
h(\frac{Y_i - \beta^T x_i}{s}) = G_i + F_i + M_i + S_i + E_i,
$$
\n(5.1)

where h is a transformation^{[1](#page-96-0)}, s is the estimated standard deviation of the residuals that have been minimized prior to transformation, and the polygenic effect (G_i) and each of the random environmental effects (F_i, M_i, S_i, E_i) are assumed to be normally distributed with zero mean. For a quantitatative trait there is also the option to transform "both sides" as in the original model described in George and Elston(1987):

$$
h(Y_i) = h(\beta^T x_i) + G_i + F_i + M_i + S_i + E_i.
$$
\n(5.2)

All covariates are mean centered prior to inclusion in the likelihood (see below). If the transformation *h* is applied to the residuals as in [\(5.1\)](#page-96-1) the estimates of the parameter values in β are asymptotically unbiased and on the original scale on which on which Y_i is measured; if h is applied to both sides as in [\(5.2\)](#page-96-2) the estimates are median unbiased. The random effects are assumed to have variances σ_G^2 , σ_F^2 , σ_M^2 , σ_S^2 *and* σ_E^2 , respectively, and these variances are on the transformed scale. Thus,

$$
V[h(Y)] = \sigma_G^2 + \sigma_F^2 + \sigma_M^2 + \sigma_S^2 + \sigma_E^2.
$$
 (5.3)

For σ_F^2 to be estimable, it is often necessary to have large pedigrees or a large number of pedigrees, or both, and therefore σ_F^2 is set equal to zero by default. Variance components divided by the total variance can be interpreted as intraclass correlations (interclass in case of the marital correlation, heritability in the case of polygenic variance); it is not possible to estimate any variances to be less than zero (in the implementation, less than .0000001).

More generally, the user can add other variance components by specifying classes of individuals who share common random effects.

¹See section [5.2.2](#page-97-0) for details of the transformation implemented in this program.

For a binary trait that takes on the value 1 or 0, $\beta^T x_i$ is replaced by $\frac{e^{\beta^T x_i}}{1-\beta^T}$ $\frac{e^{\beta^T x_i}}{1+e^{\beta^T x_i}}$ (so that $\beta^T x_i$ is the logit of Y_i in [\(5.1\)](#page-96-1)) and the variances in [\(5.3\)](#page-96-3) are rescaled to sum to 1. No further transformation is allowed in view of the finding by McCulloch and Neuhaus (2011) that in logistic regression the shape of a random effects distribution has little effect on estimating its mean and variance.

5.2.2 Transformation of the Trait

A quantitative trait *y* may be transformed by:

$$
h(y) = \begin{cases} \frac{sign(y + \lambda_2)[(|y + \lambda_2| + 1)^{\lambda_1} - 1]}{\lambda_1(y_{G2})^{(\lambda_1 - 1)}} & \text{if } \lambda_1 \neq 0, \\ y_{G2}sign(y + \lambda_2) \ln(|y + \lambda_2| + 1) & \text{if } \lambda_1 = 0 \end{cases}
$$

where

$$
y_{G2} = \left[\prod_{i=1}^{N} (|y_i + \lambda_2| + 1) \right]^{\frac{1}{N}}
$$

and $N =$ number of individuals in the sample (possibly including more than one pedigree) with complete trait and covariate values (nothing missing). This is the standardized generalized modulus power transformation (George and Elston, 1988) with power parameter λ_1 and shift parameter λ_2 . When this transformation is applied to the standardized residuals as in (5.1), λ_2 is fixed at 0.

5.2.3 Likelihood for a Randomly Sampled Pedigree

The likelihood formulation is based on the assumption of normality of the residuals and on the assumed correlational structure of the *Yⁱ* .

It should be noted that singletons (unrelated individuals) may be included in the data. Although AS-SOC counts and treats them separately for convenience, they are in fact simply one-person pedigrees with parent information missing and, as such, require no special treatment in the model.

5.2.4 Estimation of Parameters

Estimation is performed by maximizing the natural log of the likelihood numerically. If several independent pedigrees (including constituent pedigrees) are analyzed jointly, the logarithms of the likelihoods are summed overall pedigrees. The program itself determines initial estimates for the maximizing process. The user, however, may override the initial estimate of any of the parameters, or may fix them at predetermined values.

5.2.5 Models and Sample Formation

An ASSOC analysis consists of a set of models which in turn contain one or more covariates for which a test of significance may be applied. In the simplest form, each analysis contains at least two models: (1) a **baseline model** which includes, at minimum, an intercept and an individual random variance component, but may additionally include other random components and any number of covariates and (2) an **alternate model** which includes the same random components, the intercept, all baseline covariates, plus one or more covariates of interest that we wish to test. More complex models may be specified to contain either multiple versions of alternate models, or models containing multiple test covariates or some combination of the two.

Setting aside the transformation function *h*(), consider a simple model containing an intercept and three predictors, (on the logit scale for a binary trait):

$$
Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3,
$$

and further suppose that in addition to the three fixed effects represented by X_1 , X_2 and X_3 , there are three additional covariates, *A*, *B* and *C* that we would like to test, either individually or jointly. Then each of the possible combinations of test covariates may be designated by a different alternate model, for example as shown in the table below:

Thus M1 designates a model in which the covariate *A* is included in the alternate hypothesis, thereby rendering *A* as the sole test covariate. Similarly, M5 designates a model in which *A* and *C* are tested jointly. The ASSOC parameter file syntax for testing all seven combinations would be:

```
assoc {
  trait = Y \qquad # Dependent trait
  cov = X1 \qquad # First baseline predictor
  cov = X2 \qquad # Second baseline predictor
  cov = X3 # Third baseline predictor
  cov = A, models = M1, M4, M5, M7 # First test covariate
  cov = B, models = M2, M4, M6, M7 # Second test covariate
  cov = C, models = M3, M5, M6, M7 # Third test covariate
}
```
The above example would direct ASSOC to perform a likelihood ratio test for of each of the seven different alternate models M1, M2, ..., M7. The LRT for model M1 would test the effect of adding *A* into the model, the LRT for model M5 would test the effect of adding both *A* and *C* into the model, and so on.

The situation is complicated by the fact that, with respect to a given covariate, not all of the observations within a sample may be informative (i.e. because of missing covariate values). Therefore, to use the maximum amount of information for each covariate, the likelihood under the null hypothesis with respect to one covariate may differ from the null likelihood for a different covariate. (This is avoided, and the computation time is shortened, if the user imputes missing values for all covariates. Mean imputation is included as an option.)

An ASSOC analysis begins by constructing from the pedigree data a sample containing only individuals with complete information for the specific analysis to be performed, defined by the model designation. An individual has incomplete information if he/she has a missing value for either the primary trait or any of the covariates specified by either the baseline or the alternate model. If an individual is determined to have incomplete information, ASSOC will treat the individual as uninformative for all data points required by that analysis, although it will retain the relationship information for the analysis. Having constructed this baseline sample, ASSOC then finds the maximum likelihoods for the model specified in the analysis: the likelihood for the sample data is calculated once with respect to the model containing only the baseline terms (i.e., the baseline model), and once again for the specified alternate model. A new baseline sample is constructed, if necessary, for each specified alternate model.

ASSOC reports complete maximization information for each null and alternate model and, if desired, a file containing the residuals $Y_i - \beta^T x_i$ after standardization. These residuals are also available after linear transformation to make them quasi-independent (the results are calculated making the assumption that the estimate of β^T is equal to its true value). In addition, it performs a comparison test of the likelihood from each alternate model (H_1) that includes " test" covariates against that of the null (i.e., baseline) model (H_0) . If L_1 and L_0 are the maximum likelihoods under H_1 and H_0 respectively, from the same (constructed) sample, then the likelihood ratio statistic is $2[ln(L_1) - ln(L_0)]$. Under the assumption of normality of the residuals and the null hypothesis that the additional covariate(s) have no effect, this statistic is asymptotically distributed as chisquare with the number of degrees of freedom equal to the number of additional test covariates. In addition, p-values are calculated for each individual parameter in the model using its standard error obtained by double differentiation of the ln likelihood (i.e., the Wald test). These p-values are twosided for the covariate coefficients β and the transformation parameter λ_1 ; they are one-sided for all variance components. In each case the test is for the null hypothesis that the parameter is 0, except for λ_1 , where the null hypothesis is $\lambda_1 = 1$ (no transformation for the default George and Elston transformation). Note: If the transmitted allele indicator (see [3.2.7.6\)](#page-69-0) is used as a covariate, the corresponding test is a test of linkage in the presence of association, or of association in the presence of linkage to the corresponding allele (the latter assumes a valid correlation structure). This represents a transmission disequilibrium type test (TDT) for quantitative traits in extended pedigrees.

5.3 Program Input

^aASSOC does not use any information on allele frequencies or phenotype to genotype mapping that may be in the Marker Locus Description File.

5.3.1 Running assoc

A typical run of the ASSOC program may use flags to identify the file types like the following:

```
>assoc -p par -d ped -l loc
```
or, rely on a set file order like the following:

>assoc par ped loc

where par is the name of the parameter file, ped is the name of the pedigree data file and loc is the name of the optional marker locus description file.

5.3.2 The assoc Block

An assoc block in the parameter file sets the options on how to perform an analysis using ASSOC.

The following table shows the syntax for an assoc parameter which starts the assoc block.

The following table lists the parameters and attributes that may occur in an assoc block.

Notes

- 1. The title parameter also specifies the naming convention for ASSOC output files. However, the value of the out attribute (of the assoc parameter) will override the title parameter as the name of output files.
- 2. This parameter may be repeated multiple times, as needed. If a sex_code covariate is specified, the estimated effect will be for that of a female (i.e. males are coded 0, females are coded 1). This requires that sex_code has been specified as available to be used as a trait in the pedigree block, as indicated in Section [3.2.5.2.](#page-42-0)
- 3. If this attribute is not specified, ASSOC will include the covariate as part of the baseline in all models. There is no need to specify explicitly that a covariate be included in the baseline model. See section on "Models and Sampling Algorithm" in the beginning of this chapter for more details.
- 4. Treatment of val and fixed attributes for covariates and variance components is as follows:
	- (a) If, for a covariate, val is set to 0 and f ixed is set to **true**, the covariate will be "included" in the model; the effect on the analysis will be to remove all individuals for whom this covariate is missing.
	- (b) If, for a variance component (polygenic, family, marital, class effect, etc.), val is set to 0 and fixed is set to true, that random effect will be excluded from the model. This is equivalent to setting the effect to **false** (eg., $pe = false$, $fe = false$, or me = false).
	- (c) If the fixed attribute for a covariate or variance component is set to true, the attribute val must be included.
	- (d) If the fixed attribute for a covariate or variance component is set to false and the attribute val is included, this determines the initial value of the variable to be used in the maximization process. However, in the case of a variance component, val may not be set to 0 if fixed is set to false.
	- (e) If the fixed attribute for a covariate or variance component is set to false and the attribute val is not included, then the program supplies initial values for the maximization process.
- 5. Each covariate belongs to one of three categories
- (a) *baseline* : if it was listed in the analysis block but not explicitly named as a test covaraite for any model
- (b) *test* : if it was listed in the anlysis block and explicitly named as a test covariate for at least one model
- (c) *default* : if it was listed in the pedigree block but not listed within the analysis block

The batch parameter directs ASSOC to individually test each covariate in categories (b) and (c) while including those belonging to category (a) as non-test covariates within each model analyzed.

- 6. For an analysis in which one trait and ten covariates are listed in the pedigree block, for example, assuming the user does not explicitly include any single covariate but instead uses the 'batch' parameter, ASSOC will output ten pairs of maximizations (i.e, one baseline and one alternate for each of the covariates specified).
- 7. The names " Random", " Polygenic", " Family", " Sibling" and " Marital" are reserved for random effects built into the program, and may not be used here. All individuals having the same value for this categorical variable share a common effect. The parameter class_eff may be included more than once in an analysis block, in which case the total number of class effect variances estimated will equal the total number of categories in all the categorical variables.
- 8. If the value none is specified, and any individual's value is missing for any particular covariate, then that individual will be treated as uninformative for the purpose of the analysis. If mean is specified, the individual's missing covariate value will be replaced with the sample mean of that covariate (calculated on the basis of all individuals fully informative for that analysis). This can greatly improve runtime performance because the baseline model needs to be evaluated only once.
- 9. Transformation of both sides is not allowed if the primary trait is binary. By default, for a quantitative trait ASSOC will estimate λ_1 using the George-Elston transformation of the standardized difference between the trait and its expected value.

5.3.2.1 The transformation Sub-Block, applicable for continuous traits only

The following table lists the parameters and attributes that may occur in a transformation subblock.

Notes

1. An option value of none disables transformation calculations for the analysis, and an option value of george_elston means that the George-Elston transformation is used. For the George-
Elston transformation, it is strongly advised to make all the primary trait values > 1 .

- 2. The default values are $\lambda_1 = 1$ and $\lambda_2 = 0$ for the George and Elston transformation, which (if applied) would give the same result as no transformation PROVIDED that either (1) all the trait values are > 1 , or (2) all the trait values are < 1 .
- 3. Theoretically, $\lambda_1 < 0$ can never result in the trait being normally distributed, but in practice it may result in an approximate normal distribution if λ_1 is not too small. If λ_1 is allowed to decrease without bound, it is not uncommon for the other parameter estimates to take on unrealistic values. If this happens, a lower bound (e.g. -1) should be specified.
- 4. The value of λ_2 is fixed at 0 unless the **both_sides** parameter is specified.
- 5. If the both_sides parameter is not specified, then the default program behavior is to transform the difference between the trait and its expected value.

5.3.2.2 The summary_display Sub-Block

The following table lists the parameters and attributes that may occur in a summary_display subblock.

Notes

1. The meanings of the order values are as follows:

5.3.2.3 The filters Sub-Block

 \top

Notes:

1. If all is true, other filters parameters do not apply.

2. May be specified in conjunction with other filters parameters, in which case the results from the intersection of the filters are displayed .

5.3.2.4 The residuals Sub-Block

The following table lists the parameters and attributes that may occur in a residuals sub-block.

Notes:

1. The value Baseline may be used to refer to the implicit model given by the dependent variable and covariates common to all models.

The following are all valid assoc statements:

```
assoc_analysis
{
  trait = TRAIT1 # TRAIT1 is the primary trait
  cov = TRAIT2 # TRAIT2 is a (baseline) covariate
}
assoc_analysis
{
  title = "Analysis, Oct. 8, 2001"<br>trait = TRAIT3
   trait = TRAIT3 4 TRAIT3 is the primary trait
   cov = x1, models="A1" # x1 is a test covariate in the model "A1"
}
assoc_analysis, out=Assoc_res
```

```
\mathfrak{t}trait = TRAIT3 \qquad # TRAIT3 is the primary trait
   cov = X1, models="A1" # X1 is a test covariate in model "A1"
   cov = X2, models="A1, A2" # X2 is a test covariate in models A1 and A2
   cov = X3 # X3 is a baseline covariate, included in all models
   cov = TRAIT1 \qquad # TRAIT1 is a baseline covariate, included in all models
}
```
5.4 Program Output

ASSOC produces several output files that contain results and diagnostic information:

5.4.1 Summary Output File

The Summary Output File contains a model comparison table in which models containing one and only one additional covariate are compared against the null model. The table is filtered and sorted as specified in the summary_display sub-block. A table showing all models analyzed is also shown if omit_complete_summary is not included in the assoc block. Each record in either table contains the model name, the intercept estimate, estimates of the covariate coefficients, standard error for the test covariate estimate, Wald p-value and LRT p-value.

Example:

```
=================
   Results
=================
==============================
Analysis description
==============================
Title assoc<br>Primary Trait and Sbpd_
                             sbpd_rand10 (Quantitative)<br>Disabled
\Lambdallow averaging
Omit complete summary table Disabled
Summary table display order By LRT p-value
Summary filter Show no more than 10 results
```
======================================

Transformation configuration ======================================

Note: No transformation applied.

=======================

=======================

5.4.2 Detailed Output File

The detailed output includes:

.

- 1. A sample description
- 2. Variance components on the transformed scale.
	- σ²_G−Polygenic variance
	- σ_E^2 –Random variance
	- σ_F^2 –Familial variance
	- σ_M^2 –Marital variance
	- σ²_S−Sibship variance
	- ** any class effect variances, where each label ** indicates a category name
- 3. Coefficients
	- β_0 Intercept
	- β_j –Covariate coefficients, *j* > 0
- 4. Total variance: $V[h(Y_i)]$
- 5. "Heritability": $\sigma_G^2/V[h(Y_i)]$
- 6. Residual familial correlations (based on non-zero variance components)
	- Full Sibs $\sigma_S^2 + \sigma_S^2 + \frac{1}{2}$ $\frac{1}{2} \sigma_G^2$) / {*V* [*h*(*Y*_{*i*})]} • Half Sibs $\frac{2}{F} + \frac{1}{4}$ $\frac{1}{4} \sigma_G^2$ $\frac{1}{Y}$ $\frac{1}{Y}$ $\frac{1}{Y}$
- Parent-Offspring $\frac{2}{F} + \frac{1}{2}$ $\frac{1}{2}\sigma_G^2/\left\{V\left[h(Y_i)\right]\right\}$
- Marital (spouse): $\frac{2}{F} + \sigma_M^2$) / $\{V[h(Y_i)]\}$
- 7. Environmental intraclass correlations (based on non-zero variance components)
	- Nuclear Family: $\frac{2}{F}$ / $\left\{V\left[h(Y_i)\right] - \sigma_G^2\right\}$ • Marital (spouse): $\frac{2}{F} + \sigma_M^2$)/ $\{V[h(Y_i)] - \sigma_G^2\}$ • Full sibs (spouse): $\frac{2}{F} + \sigma_S^2$)/ $\{V[h(Y_i)] - \sigma_G^2\}$
- 8. Transformation paramters
	- λ_1 –Lambda 1
	- λ_2 –Lambda 2
- 9. The estimated variance-covariance matrix of all the estimated parameters
- 10. The partial first derivative of the natural logarithm of the likelihood with respect to each of the parameters estimated

In addition, several p-values are quoted based on the asymptotic distribution of the test statistics (likelihood ratio, Wald). p-values quoted for σ_G^2 , σ_E^2 , σ_F^2 , σ_M^2 , σ_S^2 and class effect variance components use a 1-sided test. All other p-values use 2-sided tests.

Example:

.


```
----------------------------------------------------------------------
Variance components
      Random 297.732700 20.424277 < 1.00e-07 -0.0000000304
Other parameters
Total variance 297.732700 20.424277 < 1.00e-07 0.0000000309
     Intercept 131.133772 0.836987 < 1.00e-07 0.0000000000
Covariates
     sex_code -4.010422 1.684304 0.017263 0.0000000000
----------------------------------------------------------------------
Final ln likelihood: -1813.490721
========================================================================
VARIANCE-COVARIANCE MATRIX foo_na_u24c without test covariates
========================================================================
---------------------------------------------------------------------
Random Total variance Intercept sex_code
---------------------------------------------------------------------
Random 417.151076 417.151076 -0.000155 0.000000
Total variance 417.151076 417.151076 -0.000155 0.000000
Intercept -0.000155 -0.000155 0.700548 0.000000
sex_code 0.000000 0.000000 0.000000 2.836879
.
.
.
```
Chapter 6

DECIPHER

DECIPHER obtains, for different types of analysis units, maximum likelihood estimates of frequencies of all possible haplotypes for autosomal or X-linked markers. The analysis units may be random individuals from the population, an individual representative of a constituent pedigree, the set of founders in a constituent pedigree, or a pooled DNA sample. In the case of members of constituent pedigrees, genotypes of other pedigree members are used to infer phase for ambiguous individuals, which improves the haplotype frequency estimates over those obtained using unrelated individuals. Haplotype frequencies can be estimated separately from different groups of analysis units that are specified by the user. A likelihood ratio test and a permutation test are provided to compare haplotype frequency distributions amoung groups.

Decipher automatically removes from analyisis markers with no data or only a single allele . In addition, the user may specify that markers with minor allele frequency less than a specified value be removed.

It can determine marker blocks by either the four gamete rule or linkage disequalibrium. The user may also specify that haplotype domains be defined by a sliding window

6.1 Limitations

Genotypes of other pedigree members can be used to infer phase for ambiguous individuals only for non-recombinant regions (i.e., no recombination is observed in the pedigree between those markers). Memory constraints may be encountered in situations where a large fraction of markers is missing, or when a large number of markers (more than 25) is haplotyped. Finally, markers in the haplotype region must be codominant, and family information and pools may not be used in the case of Xlinked markers.

6.2 Theory

6.2.1 Haplotype Frequency Estimation

The approach incorporates a variety of data types, including unrelated individuals, sets of related individuals (i.e., families), and pooled samples, or combinations of these data types. Maximum

likelihood estimates of haplotype frequencies can be obtained from pooled DNA using a form of the expectation-maximization (EM) algorithm developed expressly for that purpose (Quade et al. 2005; Ito et al. 2003; Wang et al. 2003). The key feature is the recognition that each of the other types of data can be considered a special case of pooled data. For example, unrelated individuals can be considered as pools of one individual; sets of founders in a constituent pedigree can be considered as pools of *f* individuals, where *f* is the number of founders in the pedigree. To allow combinations of the data types and to allow variation in the number of founders per pedigree, we have extended the usual EM algorithm to the situation where there are different numbers of individuals in each unit.

To estimate population haplotype frequencies, each analysis unit must come from a random set of individuals or pedigrees. For pedigree data the user can specify analysis units of single individuals, all founders or single representatives for each constituent pedigree. To choose single representatives as the analysis unit, the user may designate a single representative from each constituent pedigree or, if no individual is indicated for a particular constituent pedigree, the program will randomly select one individual out of those individuals in the pedigree with the most marker genotypes available (i.e., we are assuming genotypes are missing at random).

The form of the EM algorithm for pooled data is as follows. Suppose we are given *n* pools and each pool contains *k* individuals. The total number of markers is *m*. In this description, we primarily focus on single nucleotide polymorphisms (SNPs) with alleles encoded as 0 or 1; however, DECIPHER allows more than two alleles per locus. For each pool, at each marker position, we are given the number of 0s and the number of 1s. The sum of these two numbers is 2*k* because each individual provides 2 alleles and there are *k* individuals in each pool. The input data can be represented by a nonnegative integer matrix *M* of size *n* x *m*, where the $i - th$ row, M_i , represents the *i*-th pool and the *j* − *th* column, *M*_{*j*}, represents the *j*-th SNP, where $1 ≤ i ≤ n$ and $1 ≤ j ≤ m$. Each entry, *M*_{*ij*}, is an integer representing the number of copies of a particular allele in pool *i* at SNP *j*. The value of each entry is thus an integer in $[0, 2k]$. For *m* diallelic markers, there is a total of $T = 2^m$ possible haplotypes. Let h_t denote the *t*-th haplotype and let f_t denote its population frequency for $0 \le t \le T$. Let $H = \{h_t : 0 \le t \le T\}$ and $F = \{f_t : 0 \le t \le T\}$ be the corresponding set of all haplotypes and the set of haplotype frequencies, respectively. For a given pool M_i , let H_i denote the set of all possible haplotype assignments for *Mi*. , i.e., each element ∆ of *Hⁱ* contains 2*k* haplotypes for the *k* individuals in pool *i*. Under the assumption of Hardy-Weinberg proportions, and assuming that all the individuals are independent, the likelihood for the proportions given the data can be expressed as

$$
P(M, F) = \prod_{i=1}^{n} \sum_{\Delta \in H_i} P(\Delta).
$$
\n(6.1)

The standard EM algorithm starts with an initial assignment of the haplotype frequencies for *F*. During the E step, the expected number of each haplotype is calculated under the assumption that the haplotype frequencies are known, and during the M step the haplotype frequencies are updated according to the haplotype counts calculated in the previous E step. The two steps are iterated until convergence, defined as the minimum difference between haplotype frequencies in successive iterations being less than a small number, ε , which is specified by the user. To ensure that a global maximum is reached rather than a local maximum, the user can specify the number of different starting points that will be used. DECIPHER will obtain maximum likelihood estimates for each of this number of randomly selected starting points, and the set of estimates corresponding to the largest likelihood will be displayed. We have modified this algorithm so that the value of *k* can

differ for each pool. Note that for a pool that consists of a single male with X-linked data, *k* equals 1/2; however, in this instance the haplotype is always known with certainty.

For pedigree data, we use descent graphs to identify compatible haplotypes for a particular individual in the pedigree consistent with the observed data in the pedigree. We assume all markers are in a region with no observed recombination within the pedigrees. Using the method of descent graphs described by Sobel and Lange (1996), we can identify all possible allele states at each locus for each individual. A complete list of all possible haplotype states for each individual can then be obtained by taking the Cartesian product of the possible allele states at each locus. The possible founder haplotypes are linked through the descent graphs, such that sets of founder haplotypes that are simultaneously consistent with the observed data can be obtained. These sets of possible haplotypes, *Hi* , are then used in equation [6.1](#page-117-0) above.

There are several types of information that can be obtained. First, haplotype frequencies can be estimated for specified sets of individuals or pools. The user has the option of partitioning the individuals or pools into groups representing different subpopulations (e.g., case-control groups, ethnic groups, etc) and obtaining haplotype frequencies separately for each group. Second, we can obtain a list of all possible non-recombinant combinations of haplotypes for each individual or pool (with the constraint of < 30 markers). Third, we can obtain a list of the most likely combinations of haplotypes for each individual or pool, together with the posterior probability of each, based on population data. Because these lists can be quite large, particularly when there is a large number of markers and/or alleles, separate thresholds can be specified for displaying the lists of haplotypes and most likely haplotype combinations. Only haplotypes with an estimated frequency, or haplotype combinations with a posterior probability, greater than these respective thresholds will then be displayed. In the case where only the most likely haplotype combinations are requested, more than one haplotype combination will be returned if they have the same (maximum) posterior probability.

6.2.2 Likelihood Ratio Test

A likelihood ratio test is available to compare the distribution of haplotypes between groups (e.g., cases versus controls). Assume we have *N* groups, and we have estimated haplotype frequencies separately for each group and for the whole sample combined. Assume there are h_i haplotypes with frequency p_{ij} for haplotype *i* in group *j*. For the likelihood ratio test, the null hypothesis is H_0 : $p_{i1} = p_{i2} = ... = p_{in}$, versus the alternative hypothesis, $H_A: p_{ij} \neq p_{ik}$ for at least one haplotype *i*, and at least one pair of groups *j* and *j'* . The likelihood is maximized under these two conditions (i.e., forcing p_i to be the same for all *j* versus allowing them to be different). The likelihood ratio (LR) is then formed, and -2ln(LR) asymptotically follows a chi-square distribution with $(n-1)(h_T-1)$ degrees of freedom, where h_T is the number of different haplotypes in the whole sample. This asymptotic distribution is conservative when there are rare haplotypes, and is not recommended under those circumstances. Therefore, we also provide a method for obtaining an empirical p-value for the LR test statistic. This is obtained by sampling permutations of the group assignment (e.g., case-control status), and recomputing the LR test statistic for each permutation. On the assumption of exchangability, the empirical p-value is determined from the sample permutations as the number of permutations for which the LR test statistic exceeds the observed LR test statistic, divided by the total number of permutations.

6.2.3 Haplotype Block Determination

6.2.3.1 Four Gamete Rule

Haplotype block determination may be done by the four gamete rule as described in Wang et. al. 2002. The four gamete rule applies only if all markers in the region of interest are diallelic. A recombination between two diallelic markers is inferred by the four gamete rule if the frequency estimates of all four possible haplotypes formed by those markers exceed some user supplied threshold. These recombinations can be used to determine haplotype blocks as follows.

Haplotype frequencies for the first two markers in the region are estimated using the EM algorithm. The four gamete rule is applied to these frequency estimates using the user supplied threshold. If a recombination is found, the first marker is discarded, and the process begins again with the second and subsequent markers. If, however, no recombination is found, the first and second markers form the beginning of a block, and the procedure continues by successively pairing the third marker with the first and second markers. If a recombination is found as a result of either of these pairings, the first and second markers constitute a block, and the search for the next block begins with the third and fourth markers. Otherwise, the third marker is added to the incipient block, and the fourth marker is paired with each of the first three. The process continues in this fashion until the end of the region is reached. Blocks consisting of only one marker are discarded.

6.2.3.2 Linkage Disequilibrium

Linkage disequilibrium may be used to determine blocks by sequentially computing D' (Lewontin's LD measure) between pairs of consecutive markers. If D' exceeds the threshold for the first two markers, they constitute the beginning of a block. If D' between the second and third exceeds the threshold, the third marker becomes part of the block. This process continues until a D' is found that does not exceed the threshold, at which point the current block ends, and the process continues. When a D' is found that exceeds the threshold, a new block is begun.

6.3 Program Input

6.3.1 Running decipher

A typical run of the DECIPHER program may use flags to identify the file types like the following:

>decipher -p par -d ped -l loc -g gen

or, rely on a set file order like the following:

>decipher par ped loc

where par is the name of the parameter file, ped is the name of the pedigree data file, loc is the name of the locus description file and gen is the name of the genome description file. Note that the locus description and genome description files are optional.

6.3.2 The decipher Block

A decipher block in the parameter file sets the options on how to perform an analysis using DE-CIPHER.

The following table shows the syntax for a decipher parameter which starts the decipher block.

The following table lists the parameters and attributes that may occur in a decipher block.

- 1. May be specified more than once.
- 2. If the attributes first and last are not specified, the region parameter is used to name a region in the genome description file. Markers to be analyzed and their order are then as specified in the genome description file and the marker order in the data files is ignored. Distances between the markers are ignored by DECIPHER.
- 3. If the attributes first and last are specified, the region parameter is used to assign a name to the region whose first marker is given by the attribute first, and whose last marker is given by the attribute last (The genome description file is ignored in this case). Markers analyzed are given by the range of markers named by first and last. If the data file uses character delimited records, markers named by the marker parameter in the pedigree block of the parameter file come first, in the order they are given, followed by markers specified by the marker_list parameter, again in the order given, unless a marker locus description file is given, in which case the marker order in that file is the marker order used in the analysis. If the data file uses column delimited records, the marker order is as given in the parameter file pedigree block unless a marker locus description file is given, in which case the marker order in that file is the marker order used in the analysis. The region parameter does not apply if data sub-block parameter, analysis_unit, equals pool.
- 4. Applicable only if pop_freq or most_likely_combinations in the tasks sub-block is set to true. Starting points shown in the dump file output are generated by choosing random phase probabilities for each pool and then calculating the haplotype frequencies. To display all haplotype frequency estimates, specify cutoff to be 0.
- 5. This sub-block is required when the data are pooled.

6.3.2.1 The filters Sub-Block

The following table lists the parameters and attributes that may occur in a filters sub-block.

- 1. This parameter does not apply if the data sub-block parameter analysis_unit equals pool.
- 2. This parameter only applies if markers are diallelic.
- 3. The user must supply a marker locus description file to use this feature.

6.3.2.2 The blocks Sub-Block

The following table lists the parameters and attributes that may occur in a blocks sub-block.

- 1. If value is true, for each region the specified analysis is performed on a haplotype domain consisting of the first marker in the region and the next n - 1 markers, where n is the value of the width attribute. The analysis is repeated with a haplotype domain consisting of the second marker and the next n - 1 markers in the region. This process is continlued until the window consists of the last n markers in the region.
- 2. This parameter does not apply if the data sub-block parameter analysis_unit equals pool.
- 3. This parameter only applies if markers are diallelic.
- 4. The threshold attribute of the four_gamete_rule parameter may not be less than ten times the EM algorithm convergence criteriun (specified with the epsilon parameter).
- 5. The user must supply a marker locus description file to use this feature.

6.3.2.3 The data Sub-Block

The following table lists the parameters and attributes that may occur in a data sub-block.

- 1. For the analysis_unit parameter:
	- (a) If this parameter is set to **each individual**, all individuals will be used in the estimation of haplotype frequencies, and they will be assumed to be independent.
	- (b) If this parameter is set to family_rep, one genotyped person per constituent pedigree will be used, and familial information will be considered in determining possible haplotype combinations (in this case diplotypes) for that person. Singletons will be treated as if the parameter were set to each_individual.
	- (c) If this parameter is set to family_founders, the set of founders in a constituent pedigree will be considered as a group in determining possible haplotype combinations. If there are no genotyped founders in a constituent pedigree, or if partition information among founders is inconsistent (all founder partition values must be valid and all must have the same values to be considered consistent, see [6.3.2.3.1\)](#page-128-0), the constituent pedigree is treated as if the analysis_unit parameter were set to family_rep without a family_rep parameter being specified. Singletons are treated as if the parameter were set to each_individual.
	- (d) If this parameter is set to pool, each record in the data file is treated as a pool of genetic material as specified in the pools sub-block.
	- (e) If the value of analysis_unit is family_rep or family_founders and a Mendelian inconsistency is detected in a constituent pedigree at a particular locus, all members of the constituent pedigree are treated as if they had missing values for that locus.
- 2. If no variable is specified, the program will arbitrarily pick a genotyped individual in each constituent pedigree from among those with the most genotyped loci in the haplotype region.
- 3. The same trait or covariate may not be used as a value for both the family_rep and partition parameters.
- 4. The program looks at the values of each individual in the constituent pedigree for the family_rep variable. If no individual matches the family_rep_value, then the constituent pedigree is not used in the analysis. If only one matches the family_rep_value, the person with that value is the family representative (person whose haplotype combinations are used in the analysis). If more than one individual in a constituent pedigree has this value, the program will arbitrarily pick from among the designated individuals a genotyped individual with the most genotyped loci in the haplotype region to be the family representative.
- 5. This sub-block may appear no more than twice per analysis block and each partition subblock in an analysis block must have a unique value. If this sub-block is not specified, all

analysis units will be treated as coming from a single population. All analysis units having the same value for this variable belong to the same subpopulation (group). When the value of analysis_unit is family_rep, the family representative is determined first and that representative's partition values are used. If no representative with valid subpopulation (group) values can be found, the constituent pedigree is skipped.

- 6. If the value of analysis_unit is family_founders, partition values of the constituent pedigree founders must be consistent (as defined in note [1\)](#page-127-0) or the constituent pedigree will be skipped. The order in which the partitions are listed is significant. See note [4](#page-133-0) of the tasks sub-block for details.
- 7. This sub-block is required if analysis_unit is pools.

6.3.2.3.1 The partition Sub-Block

The following table lists the parameters and attributes that may occur in a partition sub-block.

Notes:

- 1. If a value is not given for this parameter, the group (subpopulation) name is the same as sub_pop_value. This parameter may be repeated as needed but sub_pop and sub_pop_value must be unique within a partition. If no valid groups (subpopulations) are specified, then every distinct value of the partition variable found in the data file (except the missing value), will designate a group (subpopulation).
- 2. The missing value code for the partition variable may not be specified as a sub_pop_value.
- 3. This parameter may be repeated as necessary.

6.3.2.3.2 The pools Sub-Block

The following table lists the parameters and attributes that may occur in a pools sub-block.

Notes:

- 1. This parameter designates the pool size for all pools for which pool_size_trait is not specified.
- 2. For a given record, if a value for pool_size_trait is given, it takes precedence over pool_size. Values are rounded to the nearest integer.
- 3. At least two locus sub-blocks must be given. A Genome Description File is not required or used when the value of the data sub-block parameter, analysis_unit, is pool. Map order is assumed to be the order in which the locus sub-blocks are given.

6.3.2.3.2.1 The locus Sub-Block

The following table lists the parameters and attributes that may occur in a locus sub-block..

- 1. At least one allele parameter must be included for each locus sub-block.
- 2. The value of this variable is interpreted as the pool allele probability for the allele named by the allele parameter.
- 3. last_allele must not have a trait associated with it. Its probability is one minus the sum of the probabilities of the other alleles in the locus.

6.3.2.4 The tasks Sub-Block

The following table lists the parameters and attributes that may occur in a tasks sub-block.

- 1. To display all haplotype frequency estimates, specify a cutoff of 0.
- 2. A haplotype combination is a group of haplotypes that are consistent with the genotypes of the analysis_unit over the haplotyping region. If the value of analysis_unit is each_individual or family_rep, a haplotype combination is synonymous with diplotype, but if analysis_unit is family_founders or pool, there may be more than two haplotypes to a haplotype combination.
- 3. To display all haplotype combination posterior probabilities, specify a cutoff of 0.
- 4. If two partition sub-blocks are specified, designate the first partition listed as *Partition 1* , and the second as *Partition 2* . Likelihood ratio tests are performed across the subpopulations (groups) of Partition 1 for each of the subpopulations (groups) in Partition 2, i.e., *Partition 1* is the " *inner* " partition and *Partition 2* is the " *outer* " partition. At least two subpopulations (groups) must be specified in the first partition sub-block to do this test.

The following are all valid decipher analysis blocks:

```
decipher
{
  region = "chrom 1"}
# In this next example the partition parameter is used to indicate case / control
# status.
decipher
{
   region = Chr12 # Quotes not required since the region name
                           # does not contain spaces
   data
   \mathcal{F}analysis_unit = each_individual # Do not use family information in determining
                                         # possible haplotypes.
      partition = affection_status
      {
          sub_pop = my_cases, sub_pop_value = 1
          sub_pop = my_controls, sub_pop_value = 0
      }
   }
}
# In following example, the partition parameter is used twice to partition on both
# case / control status and ethnic group.
decipher, out = "run 1"
{
   title = "1st run"
   region = "chrom 14"<br>epsilon = .0001
                         # End EM algorithm when differences in frequency estimates for
                         # successive iterations are less than .0001 for all haplotypes.
   starting_points = 3 # Run EM algorithm 3 times with a different set of
                         # starting points each time.
   data
   {
      analysis_unit = family_rep
      family_rep = T1, \qquad # Values for this trait designate one genotyped individual
                                  # per family whose possible haplotype combinations are
                                  # use in the analysis.
      family_rep_value = 1 # Haplotypes to be determined for genotyped individuals
                                  # whose value for trait, T1, equals 1.
      partition = affection_status
      \sqrt{2}sub\_pop = my\_cases, sub\_pop\_value = 1sub_pop = my_controls, sub_pop_value = 0
      }
```

```
partition = ethnicity
      {
          sub\_pop = 'african American', sub\_pop_value = 1sub\_pop = caucasian, sub\_pop_value = 0}
   }
   tasks
   {
     pop_freq = true, cutoff = .1 # Show only haplotype frequency estimates greater
                                      # than or equal to 0.1.
     likelihood_ratio_test = true
      compute_empirical_pvalue = true, permutations = 1000
   }
}
# This example illustrates the use of pooled DNA.
decipher, out=analysis1
\cdotepsilon = .000001
 data
  {
   analysis_unit=pool
   pools
    \left\{ \right.pool_size = 4 # 4 haplotypes (2 persons) per pool.
      locus = M1 # 1st locus of the haplotype region.
      {
       allele = A, trait = T1 # Variable T1 of pedigree data contains probabilities for allele A.
                                 # allele A.
        last_allele = a # probability of allele a is 1 - value of T1.
     }
      locus = M2 # 2nd locus of the haplotype region.
      {
        allele = A, trait = T2
        last_allele = a
     }
   }
 }
  tasks
  \left\{ \right.pop_freq = false
   all_possible_combinations_table = true
   most_likely_combinations = true, cutoff = .0001
    likelihood_ratio_test = false
 }
}
```
6.4 Program Output

DECIPHER produces four types of output files that contain results and diagnostic information:

6.4.1 Summary Output File

Contains results pertaining to the whole data set, specifically haplotype frequency estimates, likelihood ratio test results and empirical p-values.

Example:


```
M1 M2 M3 M4 M5 M6 M7 M8 M9
                     Haplotype Frequency Estimates
Note: Haplotypes listed have estimated frequencies greater than or equal to
      the cutoff or have the greatest frequency estimate.
Haplotype Frequency
--------- ---------
1-1-2-1-1-2-1-1-2 0.181216
1-1-1-2-1-2-2-1-2 0.178580
1-1-2-2-1-2-2-2-2 0.176715
1-2-1-1-1-2-2-2-1 0.176348<br>1-2-2-2-1-1-1-2-2 0.169943
1-2-2-2-1-1-1-2-2 0.169943<br>1-1-2-1-1-2-1-1-1 0.00383315
1-1-2-1-1-2-1-1-11-2-1-1-2-1-2-2-1 0.00340491
1-1-1-2-1-1-2-2-2 0.00308505
1-2-2-2-2-1-1-1-2 0.00307154
2-1-1-2-2-2-1-2-2 0.00249970
1-1-1-2-2-1-2-2-1 0.00249657
2-1-1-1-2-2-2-2-1 0.00242523
2-1-2-1-1-2-1-1-2 0.00234241
1-1-2-2-1-1-1-1-1 0.00202179
.
.
.
2-1-2-1-2-1-2-2-1 0.000104205
                       ---------
Total 0.999876
Ln likelihood -3980.05
```
6.4.2 Detailed Output File

Contains results on an analysis unit basis, specifically possible and most likely haplotype combinations. Note: in the following output, "haplotype combination" refers to a group of haplotypes that are consistent with the genotype of an analysis_unit. In the following example, the term is synonymous with diplotype.

Example:

```
==========
Analysis 1
==========
Markers in order:
 M1 M2 M3 M4 M5 M6 M7 M8 M9
```
Most Likely Haplotype Combinations

Note: Haplotype combinations listed have estimated probabilities greater than or equal to the cutoff or have the greatest probability estimate.

. .

Chapter 7

FCOR

FCOR can estimate multivariate familial correlations, and their asymptotic standard errors, without any distributional assumptions other than the existence of first and second moments. This can be done for all pair types available in a set of pedigrees with no marriage rings or loops. FCOR also estimates the equivalent count of independent pairs that could theoretically have been used to obtain the same standard error for each correlation. Familial correlations for both subtypes (sex-specific) and main types (ignoring sex) are estimated, together with their corresponding asymptotic standard errors. The variance-covariance matrices of the estimated correlations are calculated and a test for homogeneity of correlations among subtypes can be performed.

7.1 Limitations

Further analysis, such as adjusting for covariates, is not supported. Standard errors are based on asymptotic theory and in some cases may not be estimable.

7.2 Theory

The theory underlying all the calculations performed by FCOR is given in Keen and Elston (2003) and Matthew et al (2011).

7.2.1 Relative Pairs and Treatment of Missing Data

For each type of familial correlation, FCOR uses all pairs of relatives where both members have data on at least one trait in common. All other pairs of that type are excluded from the calculations and output. In other words, cross-correlations are not calculated for any pairs that do not have data for a common trait.

We call relative pair types that depend on individuals' sexes *subtypes*, and those that do not are called *main types*.

7.2.2 Correlations

Consider the *N* pairs of the observations of a particular type or subtype in the sample as a set of random two-element vectors $\{(x_i, y_i)\}_{i=1}^N$ $\sum_{i=1}^{N}$. These vectors are not assumed to be independent or uncorrelated, but the structure of the pairwise correlations among them is known via the pedigree structure. The pedigree correlation between the two random variables x_i and y_i is consistently estimated from a random sample of pedigrees by

$$
r_{xy} = \frac{\sum_{i=1}^{N} w_i (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{N} w_i (x_i - \overline{x})^2 \sum_{i=1}^{N} w_i (y_i - \overline{y})^2}}
$$
(7.1)

where $\bar{x} = \sum_i w_i x_i / \sum_i w_i$ and $\bar{y} = \sum_i w_i y_i / \sum_i w_i$ for arbitrary non-negative weights $\{w_i\}$. These weights are chosen to minimize the variance of the correlation being estimated, but with the restriction that using them to form confidence intervals leads to confidence intervals with the appropriate coverage (see Matthew et al 2011).

The pedigree correlation *rxy* can represent either an interclass correlation (if two classes of persons are involved) or an intraclass correlation (if only one class of persons is involved), for either the same trait or different traits. For example, suppose *rxy* represents an interclass correlation between a trait measured on a woman and a trait measured on her daughter's son. Then we can let the random variable *x* denote the woman's trait and the random variable *y* denote the trait on one of her daughter's sons. In this way, grandmother is adopted as one class and daughters' sons as another class. Given a random sample of pedigrees, the pedigrees are scanned to produce *N* pairs from the two classes whereby for the *i*th pair, x_i equals the value of a woman's trait and y_i equals the value of a trait of one of the woman's daughter's sons. If a woman's daughter has more than one son, then there will be pairs that share the observation of the same grandmother – for which an accounting must be made when calculating the asymptotic standard error. Moreover, a sibling correlation will also need to be accommodated as well. If a woman has more than one daughter, and each has at least one son, then a cousin correlation will need to be accommodated for cousin pairs who share a common grandmother. The situation becomes even more complex when pedigrees contain, for example, pairs of grandmothers as sisters. Note that one or more of the correlations needed to calculate a standard error may not be estimable.

A special case in pedigrees is that of intraclass correlations. These correlations are defined and estimated with respect to, for example: siblings; cousins; brother/brother; and female-cousin/female-cousin. The intraclass correlations are not necessarily restricted to the same random variable, or trait. In the situation of relating different random variables with members of the same class of individuals, the correlations are referred to here as intraclass cross-correlations. All possible pairs within a class of individuals are formed with the random variable *x* representing one trait and the random variable *y* representing the other trait measured on a different member of the same class.

The user can specify the largest number of generations to be considered when choosing the classes for which correlations are to be calculated. If this is unspecified (rather than being left at its default value of 2), FCOR will examine the pedigree structure and then decide for itself what pedigree

correlations can be calculated for a given random sample of pedigrees (for large pedigrees this calculation can consume a lot of computer time). Thus correlations that are not calculated are those that cannot be adequately estimated from the sample (a minimum of three pairs must be available to estimate any correlation).

7.2.3 Asymptotic Standard Errors of Correlations

The asymptotic standard error of a given correlation is estimated by using a second-order Taylor series expansion and replacing all correlation parameters with their respective estimates. If a required correlation is not estimable, it is replaced by zero or the user can suppress the calculation of such a standard error.

7.2.4 Equivalent Pair Count

The equivalent pair count for a specific familial correlation coefficient estimate is the estimated number of independent pairs of observations that would have a standard error the same as the value estimated for the specific familial correlation. Letting *r* denote the value of the correlation and *s* the estimate of its standard error, the equivalent count is estimated by

$$
EquivalentCount = \frac{1}{2} \left[N_0 + \sqrt{N_0^2 + \frac{22(1 - r^2)}{s^2} r^2} \right],
$$
\n(7.2)

where $N_0 = 1 + (1 - r^2)^2 / s^2$.

7.2.5 Test for Homogeneity of Correlations among Subtypes

This is a test of the hypothesis that all subtypes within a main type have the same correlation.

The main types are grouped by non-sex specific relationship type. For example, the SELF main type relationship contains two subtypes – male self and female self. As another example, the PARENT:OFFSPRING main type has four subtypes – father:son, father:daughter, mother:son, and mother:daughter.

Subtype correlations are always computed first, and then, if requested, main type correlations are calculated by recalulating the correlations ignoring sex. Chi-square statistics and p-values are calculated to test homogeneity of correlations among the subtypes within each main type. Under the null hypothesis of homogeneity, if only one dependent variable is being analyzed, the test statistic has an approximate chi-square distribution with degrees of freedom equal to the number of subtypes minus one. If multiple dependent variables are being analyzed, the test of homogeneity includes homogeneity of all possible subtype correlations. Thus, if there are *k* subtypes on *p* traits, then the number of degrees of freedom is $(k-1)p^2$ for interclass correlations, and $(k-1)p(p+1)/2$ for intraclass correlations.

7.2.6 P-values for Correlations

Let *r* denote the estimate of the correlation ρ . P-values for testing $\rho = 0$ are based on Fisher's *z*-transformation $z(r) = \frac{1}{2}log_e(\frac{1+r}{1-r})$ and the equivalent count *n*. For large samples, *z* is normally distributed with mean $\frac{1}{2}$ *log*_{*e*} $\left(\frac{1+\rho}{1-\rho}\right)$ $1-\rho$), and variance $\frac{1}{n-3}$ for an interclass correlation and $\frac{1}{n-\frac{3}{2}}$ for intraclass correlation. For this purpose, all cross-correlations are taken to be interclass.

7.3 Program Input

7.3.1 Running fcor

A typical run of the FCOR program may use flags to identify the file types like the following:

>fcor -p data.par -d data.ped

or, rely on a set file order like the following:

>fcor data.par data.ped

where data.par is the name of the parameter file and data.ped is the name of the pedigree data file.

7.3.2 The fcor Block

An fcor block in the parameter file sets the options on how to perform an analysis using FCOR. The following table shows the syntax for a fcor parameter which starts the fcor block.

7.3. PROGRAM INPUT CHAPTER 7. FCOR

The following table lists the parameters and attributes that may occur in an fcor block.

- 1. The value of a trait parameter should be set to the name of a trait or covariate field either read from the data file or created by a function statement . If no valid trait parameters are listed, then all trait fields are used. Note that this can lead to long runs for highly multivariate data, and that the test for homogeneity among subtypes then considers all specified traits jointly.
- 2. The type parameter is used to specify whether to calculate correlations for relative subtypes only, for main relative types only, or for both main relative types and subtypes. If the value of type is set to **subtypes**, then correlations of subtypes will be computed. If the value of type is set to **maintypes**, then correlations of main types will be computed. If the value of type is set to both, then both correlations of subtypes and main types will be computed. The default value is subtypes.
- 3. By default, any standard error for which a required correlation is nonestimable is calculated by setting the value of that required correlation to a value of 0, and appears within [] in the output. This usually overestimates the standard error. The parameter conservative specifies that if any required correlation is nonestimable, then that standard error is not calculated. The default value for conservative is false.
- 4. The generation_limit is the largest number of steps between the pair of individuals and their closest common ancestor. For example, a generation_limit value of one would include only parent offspring, sibling and half sibling pair types. A generation_limit

value of 2 would include all first-and second-degree relationships, cousins and half avuncular pairs.

- 5. The homogeneity_test parameter is used to specify calculation of chi-squares and p-values to test for homogeneity of subtypes within main types. The default value of homogeneity_test is false.
- 6. The var_cov parameter block is used to specify options to print variance-covariance matrices of subsets of the correlations. The single attribute is used to print the matrix for each trait one trait at a time, and the joint attribute is used to print a single joint matrix for two traits (se[e7.4.5\)](#page-153-0). If no attribute is specified, then single is used as the default. The traits are specified in the var_cov parameter block as trait parameters. The amount of computation and output can be limited by using a var_cov parameter block.

7.3.2.1 The output_option Sub-Block

The following table lists the parameters and attributes that may occur in an output_option subblock.

1. If the value of sex_name is set to false, then non-sex specific names will be printed in output tables. Non-sex specific name for a pair is a name for the (non-sex specific) relationship and, additionally, one or two lists of M's (for male) or F's (for female) within square brackets $([)$ that describe their ancestry. These lists represent a sequence of sexes for the individuals that comprise a lineage connecting the individuals in the pair. Relationships that represent direct descent (that is, parent-offspring, grandparental, great grandparental, and so on) are displayed as a single list starting with the ancestor and ending with the descendant. Relationships that do not represent direct descent (for example, sibling, nephew-uncle, cousin, and so on) are displayed with two lists separated by a comma. The first list begins with the first individual in the pair and terminates at the common ancestral nuclear family. The second list begins with the common nuclear family and terminates at the other individual in the pair. If the common ancestor is a parent of two half siblings (that is, the second-to-last ancestors are half siblings), then the sex of the single common ancestor is displayed between the two lists separated by two commas. The default value is true.

7.3.2.2 The var_cov Sub-Block

The following lists all parameters that may occur in a var_cov sub-block.

- 1. The value of a trait parameter should be set to the name of a trait parameter that is used in the fcor block. If no valid trait parameters are listed, then all trait fields used in the fcor block are used.
- 2. The value of correlation should be set to one of following codes or names, and can be repeated.

The following are all valid fcor statements:

```
fcor
{
}
fcor, out=test
{
   trait=TRAIT1
   trait=TRAIT2
  trait=TRAIT3
   type=maintypes
   standard_error=true
  sex_name=false
   conservative=true
   homogeneity_test=true
  generation_limit=3
   output_options
   {
     detailed_output=true
     tabular_out=true
     pairs_out=true
   }
  var_cov, single # This will calculate separate variance-
   { # covariance matrices for TRAIT1 and TRAIT2
     trait=TRAIT1 # parent:offspring type correlations.
     trait=TRAIT2
      correlation=parent:offspring
   }
   var_cov, joint # This will calculate the joint variance-
   { # covariance matrices for TRAIT1 and
     trait=TRAIT1 # TRAIT2, TRAIT1 and TRAIT3, and TRAIT2 and
     trait=TRAIT2 # TRAIT3 father:son and mother:son
     trait=TRAIT3 # correlations.
      correlation=mm # father:son
      correlation=fm # mother:son
  }
}
```
7.4 Program Output

FCOR produces several output files that contain results and diagnostic information:

7.4.1 Analysis Output File

The FCOR main analysis output file contains tables of correlations, their standard errors, used pair counts and equivalent pair counts for each pair of traits for each subtype and/or main type of relative up to 2nd generation (by default) or the generation specified by generation_limit. Addtionally, it contains pooled cross-correlations of interclass relative types and homogeneity test result when there are more than 1 trait. When homogeneity_test value is true, it also contains chi-square values, degrees of freedom and p-values.

Example:

```
============================================================================
 Tables of Correlations +/- Asymptotic Standard Errors for Sub/Maintypes
============================================================================
    Number of pedigrees : 207
    Number of traits : 2
    Trait(s) : TRAIT1 TRAIT2
```
Legend : ------ : Value is not estimable. &&&&&& : Pair count is greater than or equal to 100000. @@@@@@ : Standard error is greater than or equal to 10.0. ###### : Equivalent pair count is greater than or equal to 10000. [StdErr]: Calculated by setting at least one nonestimable required correlation to a value of 0. == . . . == Main Relationship Type : parent:offspring == Subtypes Pooled : father : son mother : son father : daughter mother : daughter Total Pairs Found = 913 --- TRAIT1 TRAIT2 INTERCLASS -- Count Correlation P-value Count Correlation P-value EqvCnt +/- StdErr EqvCnt +/- StdErr --- TRAIT1 913 0.1851 0.0000 *** 913 0.1446 0.0003 *** 645.2 +/- 0.0381 --- TRAIT2 913 0.1402 0.0061 ** 913 0.0906 0.0141 *
380.0 +/- 0.0504 733.5 +/- 0.0366 $380.0 +/- 0.0504$ $-$ Pooled Cross-Correlations ------------------------- TRAIT1 913 0.1431 0.0003 *** 636.8 +/- 0.0389 --- Test for Homogeneity of cross-correlations -- Chi-Square = 0.007990 with 1 degree(s) of freedom P-Value = 0.928776 -- Relationship Type : father(Row):son(Column) Pairs Found = 241 --- TRAIT1 TRAIT2 INTERCLASS -- Count Correlation P-value Count Correlation P-value
EqvCnt +/- StdErr EqvCnt +/- StdErr $EqvCnt$ +/- StdErr --- TRAIT1 241 -0.0011 0.9884 241 0.0355 0.6525 166.4 +/- 0.0777 163.4 +/- 0.0784 --- TRAIT2 241 -0.0494 0.5299 241 0.0195 0.8047 164.4 +/- 0.0780 163.1 +/- 0.0785 --- Pooled Cross-Correlations ------------------------- TRAIT1 241 -0.0089 0.8942

 224.2 $+/- 0.0669$ $-$

Test for Homogeneity of cross-correlations --

Chi-Square = 1.016663 with 1 degree(s) of freedom P-Value = 0.313312

```
-----------------------------------------------------------------------------
.
.
.
----------------------------------------------------------------------------
Test for Homogeneity of Correlations among Subtypes - All Traits
        ----------------------------------------------------------------------------
 Chi-Square = 23.48062 with 12 degree(s) of freedom
  P-Value = 0.023912
.
.
.
```
7.4.2 Detailed Output File

The detailed output file contains detailed tables of correlations and standard errors with different weights, used pair counts and equivalent pair counts, for each pair of traits for each subtypes and/or main type of relative up to 2nd generation (by default) or up to the generation specified by generation_limit. Generated when detailed_out value in output_option sub-block is true.

Here is a typical example of the FCOR detailed output file for the parent-offspring relationship:

```
============================================================================
  Tables of Correlations +/- Asymptotic Standard Errors for Sub/Maintypes
============================================================================
   Number of pedigrees : 207
   Number of traits : 2
   Trait(s) : TRAIT1 TRAIT2
   Legend :
     ------ : Value is not estimable.
     &&&&&& : Pair count is greater than or equal to 100000.
     @@@@@@ : Standard error is greater than or equal to 10.0.
     ###### : Equivalent pair count is greater than or equal to 10000.
      [StdErr]: Calculated by setting at least one nonestimable required
               correlation to a value of 0.
============================================================================
.
.
.
============================================================================
Main Relationship Type : parent:offspring
============================================================================
Subtypes Pooled : father : son
                  mother : son
                  father : daughter
                  mother : daughter
Total Pairs Found = 913
-----------------------------------------------------------------------------
                        TRAIT1 TRAIT2
 INTERCLASS ----------------------------------------------------------------
             Count Correlation P-value Count Correlation P-value
           EqvCnt +/- StdErr EqvCnt +/- StdErr
  -----------------------------------------------------------------------------
 PAIR_WISE WEIGHT
 ----------------
  TRAIT1 913 0.1851 0.0000 *** 913 0.1446 0.0003 ***
             593.1 +/- 0.0397 606.6 +/- 0.0398
-----------------------------------------------------------------------------
```


7.4.3 Output File of the Alternate Tabular Form

This file contains tables of correlations, their standard errors, used pair counts and equivalent pair counts in the alternate tabular form. It is generated when tabular_out value in output_option subblock is set to true.

Here is a typical example of the optional additional tabular form of output:

Relationship Type : father:son Pairs Found = 241 -- Count Correlation EqvCnt StdError P-values -- TRAIT1 - TRAIT1 241 0.0496812 190.9 0.072387 0.495490 * TRAIT1 - TRAIT2 241 0.0443138 168.6 0.077100 0.568280 TRAIT2 - TRAIT1 241 0.0258354 190.8 0.072536 0.723237 TRAIT2 - TRAIT2 241 0.0295229 168.6 0.077172 0.703905 --

7.4.4 Output File of the Smallest Number of Pairs

The FCOR pair number output prints the tables indicating, for each standard error, the smallest number of pairs used to calculate any of the required correlations. It is generated when pairs_out value in output_option sub-block is set to true

Here is a typical example of the FCOR pair numbers output tables:

```
============================================================================
 Tables of the Smallest Number of Pairs Used in
   Calculating Required Correlations for Sub/Maintypes
============================================================================
   Number of pedigrees : 207
   Number of traits : 2
   Trait(s) : TRAIT1 TRAIT2
```
[\Box] : Excluded the number of pairs for nonestimable required correlations. == . . . Relationship Type : father(Row):son(Column) Pairs Found = 241 ------------------------------------- INTERCLASS TRAIT1 TRAIT2 ------------------------------------- TRAIT1 ------------------------------------- TRAIT2 178 178 ------------------------------------- . . .

7.4.5 Variance-Covariance Matrix Output File

This file contains the variance-covariance matrix or matrices of correlation estimates. It is generated when there is a var_cov sub-block within the fcor block.

Here is a typical example of the variance-covariance matrices for TRAIT1 and TRAIT2 parentoffspring correlations:

```
==================================================
 Variance-Covariance Matrix for Correlations of
    PARENT:OFFSPRING
      with
    PARENT:OFFSPRING
    trait(s) : TRAIT1 TRAIT2 SINGLY
  **** : Value is not estimable.
  [ ] : Calculated by setting at least one nonestimable
        required correlation to a value of 0.
==================================================
 Legend :
    [Row1] PARENT:OFFSPRING - TRAIT1:TRAIT1
    [Col1] PARENT:OFFSPRING - TRAIT1:TRAIT1
  ---------------------
   \bigwedge [Col1]
  ---------------------
  [Row1] 0.0052217
  ---------------------
 The Smallest Number of Pairs Used in
  Calculating Required Correlations
  ---------------------
    \setminus [Col1]
  ---------------------
  [Row1] 86
  ---------------------
 Legend :
   [Row1] PARENT:OFFSPRING - TRAIT2:TRAIT2
```
[Col1] PARENT:OFFSPRING - TRAIT2:TRAIT2 --------------------- \bigwedge [Col1] --------------------- [Row1] [0.0062607] --------------------- The Smallest Number of Pairs Used in Calculating Required Correlations --------------------- $\qquad \qquad$ [Col1] --------------------- [Row1] 86 ---------------------

Here is another typical example of the joint variance-covariance matrices for TRAIT1, TRAIT2, and TRAIT3 father:son and mother:son correlations.

== Variance-Covariance Matrix for Correlations of FATHER:SON with MOTHER:SON trait(s) : TRAIT1 TRAIT2 TRAIT3 JOINTLY **** : Value is not estimable. [] : Calculated by setting at least one nonestimable required correlation to a value of 0. == Legend : [Row1] FATHER: SON - TRAIT1: TRAIT1 [Row2] FATHER:SON - TRAIT1:TRAIT2 [Row3] FATHER:SON - TRAIT2:TRAIT1 [Row4] FATHER:SON - TRAIT2:TRAIT2 [Col1] MOTHER:SON - TRAIT1:TRAIT1 [Col2] MOTHER: SON - TRAIT1: TRAIT2 [Col3] MOTHER:SON - TRAIT2: TRAIT1 [Col4] MOTHER:SON - TRAIT2: TRAIT2 -- \setminus [Col1] [Col2] [Col3] [Col4] -- [Row1] -0.0004526 -0.0000822 -0.0001792 -0.0000288 [Row2] -0.0000854 -0.0004590 0.0000230 -0.0001697 [Row3] 0.0003632 0.0001635 0.0000327 0.0001361 [Row4] 0.0000577 0.0002804 0.0001000 0.0001008 -- The Smallest Number of Pairs Used in Calculating Required Correlations -- \ [Col1] [Col2] [Col3] [Col4] -- [Row1] 178 178 178 178 [Row2] 178 178 178 178 [Row3] 178 178 178 178 [Row4] 178 178 178 178 -- Legend : [Row1] FATHER:SON - TRAIT1:TRAIT1 [Row2] FATHER: SON - TRAIT1: TRAIT3

The Smallest Number of Pairs Used in Calculating any Required Correlations

Chapter 8

FREQ

FREQ is a program that estimates allele frequencies and marker-specific inbreeding coefficients from marker data among related individuals with known pedigree structure and, for codominant markers, generates marker locus description files, needed by GENIBD, MLOD, and other S.A.G.E. programs.

8.1 Limitations

Maximum likelihood estimates of allele frequencies and inbreeding coefficients can only be calculated using information from pedigrees without mating rings or other loops. Any pedigrees with loops will automatically be skipped for maximum likelihood estimation. Sometimes numerical problems occur and standard errors of the frequency estimates cannot be calculated. Also, the computational time required to calculate maximum likelihood estimates increases greatly with the number of alleles at any locus.

8.2 Theory

8.2.1 Initial Frequency Estimator

FREQ begins its analysis by computing allele frequencies using only founders and singletons (unrelated and unconnected individuals) from each pedigree for all codominant marker phenotypes. These estimates are calculated by summing the number of times each allele appears and dividing by the total number of observed alleles. This estimator tends to be sub-optimal because much of the data are not used and, in many datasets, the founders are not typed.

A second estimator is provided that attempts to use marker information from non-founders and nonsingletons by assuming that they are independent. Calculation is performed the same way as for the founders, by counting the number of times each allele appears and dividing by the total number of observed alleles. These estimates can be reported directly, or combined with the founder-only based estimates by giving the founder_weight parameter a value. When the founder weight is not set, the founder and non-founder frequencies are combined by adding the number of times each allele appears in both founders and non-founders and dividing by the total number of observed

alleles from both. When founder_weight is set to a number between 0 and 1, say *w*, then a weighted average of the founder and non-founder frequencies is taken, with weights *w* and 1 − *w*, respectively. Setting founder_weight to 1 generates founder-only frequency estimates, while setting founder_weight to 0 results in non-founder-only frequency estimates.

These methods provide consistent but statistically inefficient frequency estimates which can be used for datasets that have many pedigrees with loops or markers with too many alleles for the frequencies to be computed efficiently, as well as automatically provide initial estimates for maximum likelihood estimation.

8.2.2 Maximum Likelihood Estimator

The likelihood formulation assumes that, with respect to the marker loci, the pedigrees are randomly ascertained from a single random mating population, and (unless an inbreeding coefficient is estimated, see [8.2.3](#page-157-0) below) that genotypes occur with Hardy-Weinberg equilibrium frequencies. The likelihood for the data at each marker in the whole sample is numerically maximized over possible allele frequencies to obtain the maximum likelihood estimates for that marker (Boehnke 1991). Standard errors are computed by double differentiation of the log likelihood. Those frequencies that maximize the likelihood are then reported. Non-codominant markers are fully supported, provided that the phenotype to genotype mapping is provided in a locus description file. It should be noted that singletons (unrelated and unconnected individuals) may be included in the data; they are simply one-person pedigrees with parent information missing and, as such, require no special treatment in the model.

8.2.3 Inbreeding Coefficient

FREQ also estimates, optionally, an inbreeding coefficient *f* for each marker. It is calculated by assuming, for alleles *i* and *j* with frequencies x_i and x_j , the founder genotype frequencies $x_i^2 + x_i(1-x_i)f$ for genotype ii, $2x_ix_j(1-f)$ for genotype ij and $x_j^2 + x_j(1-x_j)f$ for genotype *jj*. When the inbreeding coefficient is estimated, only maximum likelihood estimates are calculated, for both the allelle frequenceies and the marker-specific inbreeding coefficient. Standard errors are calculated by numerical double differentiation of the log likelihood.

8.3 Program Input

8.3.1 Running freq

A typical run of the FREQ program may use flags to identify the file types like the following:

>freq -p data.par -d data.ped -l m.loc

or, rely on a set file order like the following:

>freq data.par data.ped m.loc

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file, and m.loc is the name of the marker locus description file.

8.3.2 The freq Block

A freq block in the parameter file sets the options on how to perform an analysis using FREQ. The following table shows the syntax for a freq parameter which starts the freq block.

The following table lists the parameters and attributes that may occur in a freq block.

- 1. The value of this parameter is the weight given to estimates from only the founder and singleton data. It is useful when consistent (but inefficient) estimates are required from a dataset with many alleles. When not specified, the estimates labeled as "All Pedigree Members" are obtained on the assumption that all observed alleles are independent.
- 2. The value of a marker parameter should be set to the name of a marker for which allele frequencies are to be estimated. If no valid marker parameters are listed, then all markers are used.
- 3. If inbreeding is enabled, FREQ will perform two maximum likelihood estimations. The first will exclude the inbreeding coefficient; the second will include it. In addition, the output will include a two-sided likelihood ratio test of the null hypothesis $f = 0$.

8.4 Program Output

FREQ produces several output files that contain results and diagnostic information:

8.4.1 Summary Output File

The summary output file contains the following information:

- Analysis configuration
- Allele frequencies for each requested marker, including initial counts (codominant only)

Example:

======================= Freq Analysis =======================

Note: A '*' by an MLE column indicates that the corresponding maximization may not have converged. Please review the detailed output file for more information.

8.4.2 Detailed Output File

The detailed output file contains everything in the summary output file, plus complete maximization information (standard errors, final log likelihoods, derivatives) and, if an inbreeding coefficient *f* was estimated, a two-sided likelihood ratio test of the null hypothesis $f = 0$.

Example:

```
=======================
    Freq Analysis
=======================
=======================
   Configuration
=======================
 Output file: freq<br>
Founder weight: from Disabled
 Founder weight:
 Max. likelihood estimation: Enabled
 Inbreeding coefficient: Not estimated
Note: A '*' by an MLE column indicates that the corresponding maximization may not
have converged. Please review the detailed output file for more information.
=================================
    Allele frequencies 'm1'
=================================
```
-- Allele Founders only Entire dataset MLE -- 1 0.000000 0.687500 0.684802 2 1.000000 0.312500 0.315198 ================================= MAXIMIZATION RESULTS m1 ================================= --- Parameter Estimate S.E. P-value Deriv --- Alleles 1 0.684802 0.187457 -0.0000004061 2 0.315198 0.187457 -0.0000008824 --- Final Log likelihood: -4.677551 ================================= Allele frequencies 'm2' ================================= -- Allele Founders only Entire dataset MLE -- 1 0.500000 0.666667 0.607625 2 0.500000 0.333333 0.392375 ===================================== MAXIMIZATION RESULTS m2 ================================= --- Parameter Estimate S.E. P-value Deriv --- Alleles 1 0.607625 0.178317 -0.0000009799 2 0.392375 0.178317 -0.0000004529 ---

Final Log likelihood: -7.920528

8.4.3 Locus Description File

Allele frequency estimates are output into the locus description file according to the following priority:

- 1. Maximum likelihood estimates
- 2. Weighted estimates
- 3. Non-weighted/naive estimates

For example, if $skip_m = true$, and $fourm = true$, and $fourm = true$ is set to some nonzero value, then the locus description file will contain weighted estimates. On the other hand, if skip_mle is set equal to false, then the locus description file will contain maximum likelihood estimates.

Chapter 9

GENIBD

GENIBD is a program for generating identity-by-descent (IBD) sharing distributions from genetic marker locus data on family structures by a variety of algorithms tuned for various types of pedigrees. Three methods of generating IBD sharing are provided: the Single Marker IBD Analysis (single-point only), the Exact IBD Analysis (single- or multi-point), and the Simulation IBD Analysis (single- and multi-point). Control of which algorithms are used in a given analysis is provided to the user through convenient automatic switching parameters. IBD sharing distributions are generated for five types of relative pairs: sibling, half sibling, avuncular, grandparental and first cousin. The resulting output file(s) list at each location the probability of each pair sharing 0 or 2 alleles IBD, and the difference between the paternal and maternal probability of sharing 1 allele IBD, conditional on the marker data available. These files can then be read into other programs (e.g. SIBPAL) for analyses.

9.1 Limitations

IBD sharing for only five pair types can be generated:

- 1. full sib,
- 2. half sib,
- 3. grandparental,
- 4. avuncular and
- 5. first cousin.

Each constituent pedigree is treated as an independent pedigree. There are three methods currently implemented that generate IBD sharing distributions. Each method has distinct capabilities and limitations:

9.1.1 Single Marker IBD Analysis

The Single Marker IBD Analysis uses complete pedigree information at each marker individually to generate the IBD distributions for each pair of relatives at that marker. It is strictly a single-point method, and does not support pedigrees with loops.

9.1.2 Exact IBD Analysis

The Exact IBD Analysis computes the likelihood of each inheritance vector at one or several markers (including locations interpolated between markers) to generate IBD distributions for each pair of the five supported types of relative pairs at each marker. It can be used for either single- or multi-point analysis in pedigrees with or without loops. It is, however, restricted to small pedigrees due to the exponential nature of the algorithm related to the number of individuals in the pedigree. The time and space complexity of the algorithm is largely characterized by the exponent $2n - f$, the number of bits in an inheritance vector, where *n* is the number of non-founders and *f* is the number of founders in a pedigree. During parameter specification the maximum value of 2*n* − *f* may be set; any pedigree that has a value larger than the limit will use another of the analysis methods, if possible, or be skipped.

9.1.3 Simulation IBD Analysis

The Simulation IBD analysis uses a Markov chain Monte Carlo (MCMC) simulation over the space of possible inheritance vectors for each pedigree to estimate the IBD distribution for each pair of the five supported pair types at each marker, without interpolation at locations between markers. Several batches are run to ensure coverage of the state space. Generation of IBD distributions at points between markers can be accomplished by putting in markers with no data at those locations.

Also note that, since this is a simulation method, values differ between runs of the program. This method may be quite time consuming, so it should be only used when pedigrees are too large for the exact IBD analysis.

9.2 Theory

Let \hat{f}_{imj} be the probability, conditional on the marker data available, that relative pair *j* shares exactly *i* alleles IBD at marker *m*, where *i*= 0, 1 or 2. GENIBD calculates \hat{f}_{imj} for each marker locus of interest for each of five types of relative pair in the data set as follows.

Given the marker data I_m for a single pedigree at marker m

$$
\hat{f}_{imj} = \frac{P(I_m|pairj shares\,alleles\,IBD)\,P(pair\,jshares\,idleles\,IBD)}{L(I_m)}\tag{9.1}
$$

or

$$
\hat{f}_{imj} = \frac{P(I_m, pair j shares\,alleles IBD)}{L(I_m)}\tag{9.2}
$$

where $L(I_m)$ is the likelihood for the pedigree at marker m and Pr(pair *j* shares *i* alleles IBD) is the prior probability that depends on relationship alone. $L(I_m)$ does not depend on the individual pair and is thus only calculated once for each pedigree at each marker locus.

In the case of full sibs and half sibs, for $i = 1$ and pair j, the components $\hat{f}_{1mi}-$ maternal and $\hat{f}_{1mi}-$ paternal of \hat{f}_{1mj} are calculated separately, depending on the sex of the parent from whom the shared allele is descended, as follows:

$$
\hat{f}_{1mj-maternal} = \frac{P(I_m|pair\,shares\,1\,maternal allele\,IBD)P(pair\,j\,shares\,1\,maternal allele\,IBD)}{L(I_m)}
$$
\n
$$
\hat{f}_{1mj-paternal} = \frac{P(I_m|pair\,j\,shares\,1\,paternal allele\,IBD)P(pair\,j\,shares\,1\,paternal allele\,IBD)}{L(I_m)}
$$
\n
$$
\hat{f}_{1mi-matarmal} = \frac{P(I_m, pair\,j\,shares\,1\,maternal allele\,IBD)}{L(I_m)}
$$

or

$$
\hat{f}_{1mj-maternal} = \frac{P(I_m, pair \, j \, shares \, 1 \, material \, allele \, IBD)}{L(I_m)}
$$

$$
\hat{f}_{1mj-paternal} = \frac{P(I_m, pair \, j\, shares \, 1 \, patternal \, allele \, IBD)}{L(I_m)}.
$$

The difference ($\hat{f}_{1mj-maternal} - \hat{f}_{1mj-paternal}$) is reported in the GENIBD output for every marker location, denoted in the output as f1m-f1p.

The methods used to calculate these values depend on the type of analysis used.

9.2.1 Single Marker Analysis

In the case of single marker analysis, only information at a single locus is used, with $L(I_m)$ calculated using the recursive methods described in Fernando, Stricker and Elston (1993).

To calculate \hat{f}_{imj} for sib pairs, we use equation [9.1,](#page-164-0) while for other pair types we use equation [9.2.](#page-164-1) For sib pairs, we use the *counting* method suggested by Amos, Dawson and Elston (1990). To evaluate equation [9.2](#page-164-1) for other pair types, we condition upon a set of individuals in the pedigree that includes the pair and a chain of individuals connecting the pair genetically. This chain includes the parents of each member of the pair and the parents shared by any two individuals already in the chain [See Amos, Dawson and Elston (1990) for more detail.] We know that

$$
P(I_m, pair \, j \, shares \, id \, leles IBD) = \sum_{g \in G} P(I_m, pair \, j \, shares \, i \, alleles \, IBD, \, g),
$$

where *G* is the set of all possible genotype configurations of the individuals in the conditioned set. We therefore calculate $P(I_m, pair j$ shares i alleles IBD, g) for each possible genotype configuration g in G. We use the recursive methods of Fernando, Stricker and Elston (1993) to calculate the likelihood for the sections of the pedigree not in the conditioned set and reuse them for each likelihood calculation.

9.2.2 Exact IBD Analysis

The exact IBD analysis is used for both single- and multi-point analysis. It uses the exact multipoint algorithm to generate likelihoods of inheritance vectors at target locations. These likelihoods are then summed separately for inheritance vectors corresponding to a given pair sharing 0, 1, and 2 alleles IBD.

9.2.2.1 The Exact Multi-point Algorithm

The general algorithm used by MLOD and GENIBD to generate multi-point likelihoods and other statistics is called the exact multi-point algorithm. This algorithm takes a chromosomal region and generates likelihoods of all the possible inheritance patterns at each marker in the region. These likelihoods can then be combined to generate identity-by-descent statistics.

9.2.2.2 Single-point IBD Sharing

For single-point, a likelihood vector is generated for each marker of interest. For each inheritance pattern, the number of alleles shared by a given inheritance pattern can be determined by tracking which founder alleles each pair of individuals receives. By summing the likelihoods of all inheritance patterns that share a specific number of alleles IBD, and dividing by the total likelihood of the pedigree at that marker (equation [9.2](#page-164-1) above), we obtain the probability of the pair sharing that number of alleles IBD.

9.2.2.3 Multi-Point IBD Sharing

The multi-point algorithm is essentially the same as single-point. For each location of interest along the chromosome, we generate a multi-point likelihood vector incorporating all the information provided by the markers. This vector can then be summed, as in the single-point case above, to give us the multi-point probability of sharing 0, 1 and 2 alleles IBD. Although distances between markers may be specified in Haldane or Kosambi cM units (see [3.4\)](#page-74-0), once these have been translated into recombination fractions calculations proceed under the assumption of no interference.

9.2.3 Simulation IBD Analysis

The simulation IBD analysis uses a modified Sobel and Lange (1996) algorithm to generate random inheritance patterns at each marker in the state space. A multi-point likelihood for all markers is generated, again assuming no crossover interference. For each generated state, IBD values are noted. Heuristic methods are used to determine the number of states to be generated, as well as the number of batches and how much dememorization to perform.

9.2.3.1 Calculating the Amount of Simulation

By default, GENIBD itself determines the amount of simulation to perform for each pedigree. It does this by multiplying the number of individuals in the pedigree by the number of markers in the region being simulated. This number is then multiplied by several factors, one each for the number of dememorization steps per batch, the number of simulation steps per batch, and the number of batches. The default factors have been set, based upon extensive in-house testing, to the following:

These values have been found to be sufficient in most cases, but may be changed.

9.3 Program Input

9.3.1 Running genibd

A typical run of the GENIBD program may use flags to identify the file types like the following:

>genibd -p data.par -d data.ped -l m.loc -g g.map

or, rely on a set file order like the following:

>genibd data.par data.ped m.loc g.map

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file, m . loc is the name of the marker locus description file, and g. map is the name of the genome description file.

9.3.2 The genibd Block

A genibd block in the parameter file sets the options on how to perform an analysis using GENIBD.

The following table shows the syntax for a genibd parameter which starts the genibd block.

1. The out attribute controls the filenames generated by the analysis. For each region in the analysis, a separate IBD file is generated. These filenames are in the format: "*out.region*.ibd" where *out* is the value of the out attribute, and *region* is the region name.

The following table lists the parameters and attributes that may occur in a genibd block.

- 1. output_pair_types may be set to one of three values: siblings if only full sibling pairs are desired, all_sibs if both full and half sibling pairs, or relatives if all five relative pair types (sibs, half sibs, avuncular, grand-parental and cousin) are desired.
- 2. If singlepoint is selected, only data at each marker are used to calculate the IBD sharing at a marker. If multipoint is selected, all the marker data in a region are used to calculate the IBD sharing at each point, assuming no interference.
- 3. The simulation sub-block allows simulation on pedigrees for which the value of $2n f$ (see [9.1.2](#page-164-2)) exceedes the value of max_peigree. Setting the value of this parameter to always means that all pedigrees will use simulation. For example:

```
genibd, out = autism_study_01
{
  title = "Autism Study #1: IBD Results"
  region = "Chrom1"ibd_mode = multipoint
  scan_type = intervals, distance = 1.0
  simulation = always
  {
     use_factoring = true
     sim_steps = 100000
  }
}
```
9.3.2.1 The simulation Sub-Block

The following table lists the parameters and attributes that may occur in a simulation sub-block.

170

- 1. If not specified, different seeds, and hence different results, will be obtained each time a given analysis is performed.
- 2. Transitions proposed during the simulation process comprise changes to the inheritance state at specific markers or sets of markers. The parameter sim_local_marker specifies the probability that the next marker proposed for alteration is adjacent to the marker chosen immediately prior to the current proposal; otherwise a marker is chosen at random. This increases the probability of a compatible set of alterations being proposed, decreasing the time to convergence. Setting this value too high can cause a reduction in the coverage of the space due to only local changes being proposed. A lower value can result in better coverage, at the expense of time to convergence, since transitions of lower probability transitions will be proposed. The default value of 0.75 has been chosen based upon extensive in-house testing and should be sufficient for most data sets.
- 3. When calculating identity-by-descent values by simulation, it is usually unnecessary to specify the amount of simulation to be performed. GENIBD does this automatically by default for each pedigree being analyzed. However, an option to specify the amount of simulation is provided. There are two methods of doing this:
	- The first, called *factoring* , calculates the amount of dememorization, the amount of simulation, and the number of batches based upon pedigree size and number of markers in the region being simulated. It is selected by setting use_factoring to true (default). The user may set the value of base_factor (which automatically determines the values of demem_factor, sim_factor and sim_batch_factor as described in the syntax table) or may set the values of demem_factor, sim_factor and sim batch factor directly.
	- The second method uses the same number of steps and batches for every pedigree. It is used when use_factoring is set to false. Setting demem_steps, sim_steps, and batch_count parameters sets, respectively, the amount of dememorization per batch, the amount of simulation per batch, and the number of batches.

9.4 Program Output

GENIBD produces several output files that contain results and diagnostic information:

9.4.1 Genome Information File

This file includes warnings and errors produced while parsing the marker locus description file, as well as a table for each marker listing allele and genotype population frequencies, assuming Hardy-Weinberg equilibrium. If allele frequencies do not sum to 1.0, they are standardized to 1.0, so these frequencies may not be identical to those in the marker locus description files.

9.4.2 IBD Sharing Files

The IBD sharing file stores the IBD probability distribution of allele-sharing identical-by-descent between pairs of individuals at specific locations.

The IBD sharing file is generated as output from GENIBD and is used as input to other programs, such as SIBPAL. It contains the following information (see [3.5\)](#page-77-0):

- a list of the markers at which the IBD sharing distributions are generated.
- a table that contains a line for each relative pair and the probabilities of sharing 0 or 2 alleles at each marker (designated as f_0 and f_2 , respectively) and the value of the difference $f_{1m}-f_{1p}$ to support analysis of parent-of-origin effects. The table includes up to five types of relative pairs: sibling, half sibling, avuncular, grand-parental and cousin.

In the following example, a multi-point IBD sharing file is generated. Although the numerical results are different, the single-point file is similar in structure. The following is a portion of the file:

Chapter 10

LODLINK

LODLINK performs model-based lod score calculations for two-point linkage between a main trait/marker and each of the other markers in the pedigree file. The main trait/marker may be a marker or a trait that follows Mendelian transmission and has either two or three types. When a trait is used (as opposed to a marker) as the main trait/marker, output from SEGREG can be used as input. LODLINK uses the genotype/phase elimination algorithms proposed by Lange and Boehnke (1983) and Lange and Goradia (1987), together with other enhancements, to perform relatively fast exact linkage calculations.

10.1 Limitations

Pedigrees may not contain loops or marriage rings. Note that for X-linkage both the main traitmarker (see [3.2.5.4\)](#page-49-0) and the marker (see [3.2.5.4\)](#page-49-0) must be specified to be X-linked and all hemizygous males must be coded as homozygous. Be aware that for X-linkage the main result will be correct, but while we believe the results are appropriate for all the options available in LODLINK, this has not been carefully checked.

10.2 Theory

10.2.1 Computation of the Likelihood and Lod Scores

Let $T_1, ..., T_k$ be the alleles at the trait locus, $q_{T_1}, ..., q_{T_k}$ be the corresponding frequencies, $M_1, ..., M_m$ be the alleles at the marker locus, and *qM*,...,*qM^m* be the corresponding allele frequencies.

1. For autosomal and pseudo-autosomal linkage, the probability of a phased joint genotype $\frac{T_b M_c}{T_d M_e}$ where $b, d = 1, \dots, k$; $c, e = 1, \dots, m$, in the population is

$$
\psi\left(\frac{T_bM_c}{T_dM_e}\right) = C\psi(T_bT_d)\psi(M_cM_e),
$$

where

$$
C = \begin{cases} 1 & \text{if } T_b = T_d \text{ and } M_c = M_e, \\ \frac{1}{2} & \text{if } (T_b = T_d \text{ and } M_c \neq M_e) \text{ or } (T_b \neq T_d \text{ and } M_c = M_e) \\ \frac{1}{4} & \text{otherwise} \end{cases}
$$

and

$$
\psi(T_b T_d)
$$
 = probability of trait genotype in the population,
 $\psi(M_c M_e)$ = probability of marker genotype in the population.

For X-linkage, females are as above, but males are hemizygous and for them the joint genotype probability in the population is

$$
\psi(T_bM_c)=\psi(T_b)\psi(M_c),
$$

where (in SEGREG) $b = A$ or B.

2. The transmission probability for autosomal and pseudo-autosomal linkage is

$$
\tau_s \left(\frac{T_b M_c}{T_d M_e} \rightarrow T_f M_g \right) = Pr \left(\text{ parent of sex } s \text{ and genotype } \frac{T_b M_c}{T_d M_e} \text{ transmits } T_f M_g \text{ to child } \right)
$$
\n
$$
= \frac{(1 - \theta_s)(\delta_{T_b T_f} \delta_{M_c M_g} + \delta_{T_d T_f} \delta_{M_e M_g})}{2} + \frac{\theta_s(\delta_{T_b T_f} \delta_{M_e M_g} + \delta_{T_d T_f} \delta_{M_c M_g})}{2},
$$

where θ_s is the sex-dependent recombination fraction between the trait and marker loci ($\theta_s =$ θ_{male} or θ_{female}) and

$$
\delta_{xy} = \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{if } x \neq y. \end{cases}
$$

For X-linkage, the transmission probability depends on the sexes of both parent and offspring. For female parents,

$$
\tau_{female} \left(\frac{T_b M_c}{T_d M_e} \rightarrow T_f M_g \right) = Pr \left(\text{ female parent with genotype } \frac{T_b M_c}{T_d M_e} \text{ transmits } T_f M_g \text{ to child } \right)
$$
\n
$$
= \frac{(1 - \theta_{female}) (\delta_{T_b T_f} \delta_{M_c M_g} + \delta_{T_d T_f} \delta_{M_e M_g})}{2} + \frac{\theta_{female} (\delta_{T_b T_f} \delta_{M_e M_g} + \delta_{T_d T_f} \delta_{M_c M_g})}{2}
$$

for all children, the male children being hemizygous.

For male parents,

$$
\tau_{male}(T_bM_c \rightarrow T_fM_g) = Pr \left(\text{ male parent with genotype }T_bM_c \text{ transmits }T_fM_g \text{ to child }\right)
$$

$$
= (1-\theta_{male})(\delta_{T_bT_f}\delta_{M_cM_g} + \delta_{T_dT_f}\delta_{M_eM_g}) + \theta_{male}(\delta_{T_bT_f}\delta_{M_eM_g} + \delta_{T_dT_f}\delta_{M_cM_g})
$$

for female children and is irrelevant for the male children.

3. The transition probability is

$$
Tr\left(\frac{T_bM_c}{T_dM_e},\frac{T_rM_s}{T_uM_v},\frac{T_fM_g}{T_hM_j}\right) = Pr\left(\begin{array}{c} \text{Mother with genotype } \frac{T_bM_c}{T_dM_e} \text{ and Father with genotype } \frac{T_rM_s}{T_uM_v} \\ \text{have a child with genotype } \frac{T_fM_g}{T_hM_j} \end{array}\right)
$$
\n
$$
= \begin{cases} \tau_{female}\left(\frac{T_bM_c}{T_dM_e} \to T_fM_g\right) \tau_{male}\left(\frac{T_rM_s}{T_uM_v} \to T_hM_j\right) & \text{if } T_f = T_h \text{ and } M_g = M_j, \\ \tau_{female}\left(\frac{T_bM_c}{T_dM_e} \to T_fM_g\right) \tau_{male}\left(\frac{T_rM_s}{T_uM_v} \to T_hM_j\right) + \\ \tau_{female}\left(\frac{T_bM_c}{T_dM_e} \to T_hM_j\right) \tau_{male}\left(\frac{T_rM_s}{T_uM_v} \to T_fM_g\right) & \text{otherwise.} \end{cases}
$$

4. For a phased joint genotype $\frac{T_b M_c}{T_d M_e}$ of pedigree member *i*, let the separate one-locus genotypes be denoted $u_i = T_b T_d$ for the trait and $v_i = M_c M_e$ for the marker. Let y_i be the trait and m_i be the marker phenotype (discrete). Let w_{male_i} , w_{female_i} and w_i indicate the phased two-locus (i.e., joint) genotypes of the father of individual *i*, mother of individual *i*, and individual *i* respectively. The likelihood for a pedigree of *n* persons is

$$
L(\theta) = \sum_{w_1} \dots \sum_{w_n} \prod_{i=1}^n H_i,
$$

where

$$
H_i = \begin{cases} p_i(w_{female_i}, w_{male_i}, w_i) & \text{if } i \text{ has any missing data,} \\ p_i(w_{female_i}, w_{male_i}, w_i) g_{u_i}(y_i) g_{v_i}(m_i) & \text{otherwise} \end{cases}
$$

in which

$$
p_i(w_{female_i}, w_{male_i}, w_i) = \begin{cases} Tr(w_{female_i}, w_{male_i}, w_i) & if the parents of i are in the pedigree, \\ \psi(w_i) & otherwise \end{cases}
$$

 $g_{v_i}(m_i)$ = probability of marker phenotype m_i given marker genotype v_i (assumed to be always 0 or 1).

 $g_{u_i}(y_i)$ = probability (density) of trait y_i conditional on genotypes u_i and possibly other factors. These can be obtained as output from SEGREG by specifying type_prob = true in the SEGREG output_options sub-block.

Lod scores are defined as $Z(\theta) = \text{Log}_{10}L(\theta) - \text{Log}_{10}L(0.5)$.

If pedigree member *i* is a hemizygous male with joint genotype T_bM_c , then phase is irrelevant and it is only necessary to sum over the phases of females in $L(\theta)$. Let w_{female_i} and w_i indicate the phased two-locus genotypes of the mother of individual i , and individual i is male, respectively. Then for a male individual we have

$$
H_i = \begin{cases} p_i(w_{female}, w_i) & if i has any missing data, \\ p_i(w_{female}, w_i)g_{u_i}(y_i)g_{v_i}(m_i) & otherwise \end{cases}
$$

and for a female individual, we have the same H_i as given above.

10.2.2 Estimation of Parameters

When estimating the recombination fraction θ , maximum likelihood estimates of θ are obtained as the values that make the likelihood largest in the parameter space [0, 0.5]. If a larger likelihood exists for θ in the parameter space [0, 1], the corresponding estimate(s) are also given.

When estimating both the recombination fraction θ and the proportion of linked families, α , maximum likelihood estimates are obtained over the range of parameter values indicated in the output.

10.2.3 Hypothesis Tests

10.2.3.1 Maximum Lod Score Test for Linkage

If we are estimating recombination fractions with $\theta_{male} = \theta_{female}$, then the asymptotic chi-square statistic calculated is

$$
\chi_1^2 = 2[\log_e L(\hat{\theta}) - \log_e L(0.5)]
$$

and the corresponding p-value quoted is

$$
1-\Phi\left(\sqrt{\chi_1^2}\right),\,
$$

where Φ is the standard cumulative normal distribution. The upper bound of the p-value is calculated as $\frac{1}{10^{z(\hat{\theta})}}$. The p-value and upper bound are quoted only if $0 \leq \hat{\theta} < 0.5$.

If we are calculating the recombination fractions for males and females separately, the chi-square statistic calculated is

$$
\chi_2^2 = 2[\log_e L(\hat{\theta}_{male}, \hat{\theta}_{female}) - \log_e L(0.5, 0.5)]
$$

,

The corresponding p-value quoted as corresponding to this lod score is calculated on the assumption that the estimates $\hat{\theta}_{male}$ and $\hat{\theta}_{female}$ are independent, i.e. assuming that, under the null hypothesis $\hat{\theta}_{male} = \hat{\theta}_{female} = 0.5, 2 \log_e 10 \times (maximum lod)$ is distributed as $\frac{1}{4} + \frac{1}{2}$ $\frac{1}{2}\chi_1^2 + \frac{1}{4}$ $\frac{1}{4}\chi_2^2$. The upper bound of the p-value is calculated as

$$
\frac{1}{10^{z(\hat{\theta}_{male},\hat{\theta}_{female})}}.
$$

The p-value and upper bound are quoted only if $0 \leq \hat{\theta}_{male}, \hat{\theta}_{female} < 0.5$.

10.2.3.2 Cleves and Elston's (1997) Likelihood Ratio Test for Linkage

Let $L(\hat{\theta}_{male}, \hat{\theta}_{female})$ be the likelihood evaluated at the maximum likelihood estimates $\hat{\theta}_{male}, \hat{\theta}_{female}$ and $L(\tilde{\theta}_{male}, \tilde{\theta}_{female})$ be the likelihood estimated at the values $\tilde{\theta}_{male}, \tilde{\theta}_{female}$ that maximize the likelihood under the constraint $\tilde{\theta}_{male} + \tilde{\theta}_{female} = 1$. Then the asymptotic chi-square statistic calculated is

$$
\chi_1^2 = 2[\log_e L(\hat{\theta}_{male}, \hat{\theta}_{female}) - \log_e L(\tilde{\theta}_{male}, \tilde{\theta}_{female})]
$$

and the corresponding p-value quoted is

$$
1-\Phi\left(\sqrt{\chi_1^2}\right),
$$

where Φ is the standard cumulative normal distribution. If both $\hat{\theta}_{male}$ and $\hat{\theta}_{female}$, are > 0.5 no p-value is calculated.

10.2.3.3 Morton's (1956) Likelihood Ratio Test for Homogeneity of the Recombination Fraction

Let $\sum_{n=1}^n$ $\sum_{i=1}^{n} \log_e L_i(\hat{\theta}_i)$ be the maximum log likelihood over n groups of pedigrees with $\hat{\theta}_i$ estimated separately for each group, and let $\sum_{n=1}^{\infty}$ $\sum_{i=1}^{n} \log_e L_i(\hat{\theta})$ be the maximum log likelihood over the n groups with a common $\hat{\theta}$ estimated; then the asymptotic chi-square statistic is

$$
2[\sum_{i=1}^{n} \log_{e} L_{i}(\hat{\theta}_{i}) - \sum_{i=1}^{n} \log_{e} L_{i}(\hat{\theta})], \text{ with } n-1 \text{ degrees of freedom if } \theta_{male} = \theta_{female}
$$

$$
2(n-1) \text{ degrees of freedom if } \theta_{male} \neq \theta_{female}
$$

The "asymptotic p-value" is the p-value based on the statistic following a chi-square distribution.
10.2.3.4 Smith's (1963) Test for Homogeneity of the Recombination Fraction

Let θ < 0.5 be the recombination fraction in a proportion α of the families, and suppose there is no linkage in the remaining 1 -α of the families. Define the log likelihood of the i-th family as log*^e* $L_i(\alpha, \theta) = \log_e[\alpha L_i(\theta) + (1 - \alpha) L_i(0.5)]$. Under the model $0 < \alpha < 1$, and $0 < \theta < 0.5$, we test the null hypothesis $\alpha = 1$.

Let $\sum_{n=1}^n$ $\sum_{i=1}^{n} \log_e L_i(\hat{\alpha}, \hat{\theta})$ be the maximum log likelihood over *n* constituent pedigrees with α and θ *i*=1 estimated, and $\sum_{n=1}^{\infty}$ $\sum_{i=1}^{n} \log_e L_i(1, \hat{\theta})$ be the maximum log likelihood over n constituent pedigrees with α

 $= 1$ and θ estimated.

If $\hat{\theta}$ is scalar (i.e., we assume $\theta_{male} = \theta_{female}$) then the asymptotic chi-square statistic for heterogeneity versus homogeneity is

$$
\chi_1^2 = 2[\sum_{i=1}^n \log_e L_i(\hat{\alpha}, \hat{\theta}) - \sum_{i=1}^n \log_e L_i(1, \hat{\theta})], and the one side d p-value is 1-\Phi(\sqrt{\chi_1^2}),
$$

where Φ is the standard cumulative normal distribution.

If $\theta_{male} = \theta_{female}$ is not assumed, so that both $\hat{\theta}_{male}$ *and* $\hat{\theta}_{female}$ are estimated, the chi-square statistic is compared to the chi-square distribution with 2 degrees of freedom and the asymptotic p-value is "two-sided".

10.2.3.5 Faraway's (1993) Test for Linkage Under Smith's (1963) Heterogeneity Model.

The asymptotic "chi-square" for linkage in the presence of heterogeneity is

$$
2[\sum_{i=1}^n \log_e L_i(\hat{\alpha}, \hat{\theta}) - \sum_{i=1}^n \log_e L_i(0.5)],
$$

for which the p-value is obtained on the assumption that this statistic is distributed as the maximum of two independent chi-square variables, each with one degree of freedom.

If $\theta_{male} = \theta_{female}$ is not assumed, the "chi-square" statistic is assumed to be distributed as the maximum of two independent chi-square variables, each with 2 degrees of freedom, and the asymptotic p-value quoted is "two-sided".

10.2.3.6 Posterior Probability of Linkage

The posterior probability that the i-th family belongs to the linked type, given the observations, is computed as

$$
w_i(\hat{\alpha}, \hat{\theta}) = \frac{\hat{\alpha} L_i^*(\hat{\theta})}{\hat{\alpha} L_i^*(\hat{\theta}) + 1 - \hat{\alpha}},
$$

where

$$
L_i^* = \frac{L_i}{L_i(0.5)}.
$$

 $w_i(\hat{\alpha}, \hat{\theta}) > \hat{\alpha}$ indicates that the i-th family contains evidence for linkage.

10.2.4 Conditional Trait Genotype Probabilities

The table in the detailed file headed "Individual Genotype Probabilities" gives, for each pedigree member, the probabilities of having trait genotypes *bd* conditional on that member's output marker phenotype, assuming maximum likelihood estimates of the recombination fraction (or fractions, sex specific), and assuming homogeneity across pedigrees, i.e., expressing $L(\theta)$ as a function of the two locus genotypes *bc/de* (*bd* for the trait and *de* for the marker), *L(bc/de)* ,

$$
P_{bd} = \frac{\sum_{allce} L(bc/de)}{\sum_{allbd} \sum_{allce} L(bc/de)}
$$

where, by default, $bd = AA$, AB or BB as in SEGREG.

10.3 Program Input

a If an allele frequency for a particular individual is zero, then the likelihood for that individual's pedigree will be zero, and the pedigree will effectively be skipped during analysis.

*b*Both the trait locus description file and the trait genotype probability file are optional. One, but not both, may be used for LODLINK input.

10.3.1 Running lodlink

A typical run of the LODLINK program may use flags to identify the file types like the following:

>lodlink -p par -d ped -l loc -m mld (or typ)

or, rely on a set file order like the following:

>lodlink par ped loc tld (or typ)

where par is the name of the parameter file, ped is the name of the pedigree data file, loc is the name of the locus description file, tld is the name of the trait locus description file and typ is the name of the trait locus description file.

10.3.2 The lodlink Block

A lodlink block in the parameter file sets the options on how to perform an analysis using LODLINK. The following table shows the syntax for a lodlink parameter which starts the lodlink block.

The following table lists the parameters and attributes that may occur in a lodlink block.

Notes:

1. A value for either a trait or marker must be specified, but not both.

2. Linkage tests are performed according to entries in the following table, depending on the values assigned to the sex_specific and homog attributes of the linkage_tests parameter.

- 3. The default is to perform no linkage homogeneity tests. Otherwise a homog_tests sub-block must be included.
- 4. The default is to calculate lod scores for the following non-sex-specific recombination fractions: 0, .01, .05, 0.1, 0.2, 0.3 and 0.4. Otherwise a lods sub-block must be included.

10.3.2.1 The homog_tests Sub-Block

The following table lists the parameters and attributes that may occur in a homog_tests sub-block.

10.3.2.1.1 The mortons_test Sub-Block

The following table lists the parameters and attributes that may occur in a mortons_test subblock.

Notes

- 1. If no groups are specified, each pedigree (which may contain multiple constituent pedigees, all assumed to be mutually independent) is its own group.
- 2. Each pedigree must be listed in one, and only one group in the group sub-block described below.

10.3.2.1.1.1 The group Sub-Block

The following table lists the parameters and attributes that may occur in a group sub-block.

Notes:

- 1. Required if group parameter is specified.
- 2. Example:

```
lodlink
{
  model, trait = T1
  linkage_tests = false
  homog_tests
  {
```

```
smiths_test = false #explicitly set to the default value
   mortons_test = true, sex_specific = false
   {
      \text{group} = 1{
         pedigree_id = 1
         pedigree_id = 2
         pedigree_id = 3
         pedigree_id = 4
         pedigree_id = 5
      }
      group = 2
      {
         pedigree_id = 6
         pedigree_id = 7
         pedigree_id = 8
      }
   }
}
lods
\left\{ \right.option = none
}
```
10.3.2.2 The lods Sub-Block

}

The following table lists the parameters and attributes that may occur in a lods sub-block.

Notes

- 1. If none is specified, no lod scores will be calculated. If standard is specified, lod scores will be calculated for the following non-sex-specific recombination fractions: 0, .01, .05, 0.1, 0.2, 0.3 and 0.4. If specified is specified, the desired recombination fractions must be specified using male_female or average sub-blocks for sex-specific or sex-averaged recombination fractions, respectively.
- 2. Required if the option parameter is set to **specified** and the sex_specific parameter is set to true.
- 3. Required if the option parameter is set to specified and the sex_specific parameter is set to false.

10.3.2.2.1 The male_female Sub-Block

The following table lists the parameters and attributes that may occur in a male_female sub-block.

Notes:

1. Required if the theta parameter is specified.

10.3.2.2.2 The average Sub-Block

The following table lists the parameters and attributes that may occur in a average sub-block.

Notes:

1. Required if the average parameter is specified.

Example 1

Do Morton's test for linkage homogeneity between model T1 (produced by SEGREG) and each marker in the pedigree file, estimating non-sex-specific recombination fractions. For these tests the group designated "1" consists of pedigrees 1-5 and group "2" consists of pedigrees 6-7.

```
lodlink
{
   model, trait = T1linkage_tests = false
   homog_tests
   {
      smiths_test = false #explicitly set to the default value
      mortons_test = true, sex_specific = false
      {
         group = 1
         {
            pedigree_id = 1
            pedigree_id = 2
            pedigree_id = 3
            pedigree_id = 4
         }
         group = 2
         {
            pedigree_id = 5
            pedigree_id = 6
            pedigree_id = 7
         }
      }
   }
   lods
   {
      option = none
   }
}
```
Example 2

Test for linkage between marker "Mfd154" and each of the other markers in the pedigree file estimating sex-specific recombination fractions assuming linkage homogeneity. Also calculate lod scores for the following pairs of recombination fractions: male .4, female 0; male .4, female .1; male .3, female .2.

Use the title "linkage test" in the output files. Name the summary and detail output files "example2.sum" and "example2.det", respectively.

```
lodlink, out ="example2"
{
   title ="linkage test"
   model, marker = Mfd154
   linkage_tests = true, sex_specific = true, homog = true
   homog_tests
   {
      smiths_test = false #explicitly set to the default value
      mortons_test = false #explicitly set to the default value
   }
   lods
   {
      option = specified
      sex_specific = true
      male_female
      {
         theta, male = .4, female = 0theta, male = .4, female = .1theta, male = .3, female = .2}
   }
}
```
10.4 Program Output

LODLINK produces four types of output files that contain results and diagnostic information:

10.4.1 Genome Information Output File

This file lists for each marker the allele and genotype population frequencies, assuming Hardy-Weinberg equilibrium. If allele frequencies do not sum to 1.0, they are standardized to 1.0, so these frequencies may not be as described in the locus description files.

10.4.2 Summary Output File

Contains results pertaining to the whole data set. See [10.2](#page-175-0) for details regarding interpretation of these results.

Example:

10.4.3 Detailed Output File

Contains results on a per individual, per family or per group basis. See [10.2](#page-175-0) for details regarding interpretation of these results.

Example:

============================== LODLINK Analysis 1 DETAIL FILE ============================== Options Selected .
================ Main locus type trait

Main locus name the second trait Main locus name Lod scores yes Recombination Fractions Selected ================================ 0.0000 0.0100 0.0500 0.1000 0.2000 0.3000

Lod Scores By Family Non-Sex-Specific Recombination Fractions

Constituent Pedigree in Pedigree 1 Containing Member 1

Constituent Pedigree in Pedigree 2 Containing Member 1

. . .

Constituent Pedigree in Pedigree 199 Containing Member 1

Constituent Pedigree in Pedigree 200 Containing Member 1

Lod Score Linkage Test Variance-Covariance Matrices Parameter Order (Avg Recomb)

Chapter 11

LODPAL

LODPAL performs a linkage analysis based on the LOD score formulation for affected-sib-pairs (ASPs) (Risch, 1990). The current implementation is of the general conditional logistic model proposed by Olson (1999) modified to give the one-parameter model of Goddard et al. (2001). The model allows for the inclusion of all affected-relative-pairs (ARPs) and covariates or discordant sibling pairs, with the possibility of pooling unaffected relative pairs together with ARPs in the analysis.

11.1 Limitations

The current release only includes support for a single disease locus and assumes all pairs of relatives are independent.

11.2 Theory

11.2.1 Basic notation

Let the number of relative pairs be *n* .

Let *j* index the relative pair: $j = 1, 2, ..., n$.

Let f_{r0} , f_{r1} , and f_{r2} be the prior probabilities of sharing 0, 1, or 2 alleles IBD given a relative pair of type *r*.

Let

 \hat{f}_{0j} be the probability of sharing 0 alleles IBD at a given marker location, for the *j*-th pair,

 \hat{f}_{1j} be the probability of sharing 1 allele IBD at a given marker location, for the *j* -th pair, and

 \hat{f}_{2j} be the probability of sharing 2 alleles IBD at a given marker location, ffor the *j*-th pair.

These three IBD-sharing probabilities are estimated by GENIBD given the available marker data and given the pedigree relationship (i.e., type of relative pair). They may be multi-marker or singlemarker estimates. Marker is the equivalent to marker location, and need not be a measured marker. This is mainly an issue dealt with in the IBD generation phase.

The following table summarizes the various notation that has been used for the probability of sharing *i* alleles IBD between affected sib pairs at a particular locus, where λ_i is the locus-specific risk ratio or relative recurrence risk for a relative who shares *i* alleles identical by descent with an affected person:

The sibling locus-specific relative recurrence risk is given by

$$
\lambda_s=\frac{1}{4}\left[\lambda_0+2\lambda_1+\lambda_2\right]=\frac{1}{4}\left[1+2\lambda_1+\lambda_2\right]=\frac{1}{4}+\frac{1}{2}\lambda_1+\frac{1}{4}\lambda_2
$$

11.2.2 Affected Relative Pair Linkage Analysis

11.2.2.1 Two-parameter Model (Olson 1999)

The LOD score for a set of *n* independent ARPs is

$$
z = \sum_{j=1}^{n} \log_{10} \left\{ \frac{\hat{f}_{0j} + \hat{f}_{1j} e^{\beta_1} + \hat{f}_{2j} e^{\beta_2}}{f_{r0} + f_{r1} e^{\beta_1} + f_{r2} e^{\beta_2}} \right\}
$$

=
$$
\sum_{j=1}^{n} \log_{10} \left\{ \frac{\sum_{i=0,1,2} \hat{f}_{ij} e^{\beta_i}}{\sum_{i=0,1,2} f_{ri} e^{\beta_i}} \right\} = \sum_{j=1}^{n} \log_{10} \left\{ \frac{\sum_{i=0,1,2} \hat{f}_{ij} \lambda_i}{\sum_{i=0,1,2} f_{ri} \lambda_i} \right\},
$$

Here, $\beta_0 = 0$, and β_1, β_2 are estimated by maximizing the LOD score with the constraints $\beta_1 \ge 0$ and $\beta_2 \ge \log_e (2e^{\beta_1}-1)$ (i.e., $\lambda_1 > 1$ and $\lambda_2 > 2\lambda_1 - 1$).

For full sibs, $f_{S_0} = \frac{1}{4}$ $\frac{1}{4}$, $f_{S_1} = \frac{1}{2}$ $\frac{1}{2}$, $f_{S_2} = \frac{1}{4}$ $\frac{1}{4}$, giving for the *j* -th full sib pair

$$
\log_{10}\left\{4\frac{\hat{f}_{0j}+\hat{f}_{1j}e^{\beta_1}+\hat{f}_{2j}e^{\beta_2}}{1+2e^{\beta_1}+e^{\beta_2}}\right\}.
$$

For half sibs, $f_{h0} = \frac{1}{2}$ $\frac{1}{2}$, $f_{h1} = \frac{1}{2}$ $\frac{1}{2}$, $f_{h2} = 0$, giving for the *j* -th half sib pair

$$
\log_{10}\left\{2\frac{\hat{f}_{0i}+\hat{f}_{1j}e^{\beta_1}}{1+e^{\beta_1}}\right\}.
$$

In summary,

In the next sections, the subscript *i* indexing the pair and the summation over *j* will be suppressed.

11.2.2.2 One Parameter Model

Under the optimal one parameter model, the LOD score contribution of a single pair is

$$
\log_{10}\left\{\frac{\hat{f}_0 + \hat{f}_1e^{\beta_1} + \hat{f}_2(3.634e^{\beta_1} - 2.634)}{f_{r0} + f_{r1}e^{\beta_1} + f_{r2}(3.634e^{\beta_1} - 2.634)}\right\}.
$$

The constants in the above expression, $\alpha = 2.634$ and $\alpha + 1 = 3.634$, fixes the mode of inheritance to a value approximately halfway between a dominant and a recessive model and correspond to the Whittemore and Tu (1998) minmax model mode of inheritance parameter (they defined two parameters, *w*₁ and *a*, and the minmax values of these are $w_1 = 0.275$ and $\alpha = (2 - 3a)/a$. To allow more flexibility, the user may specify a different "mode of inheritance" parameter. Thus, under a generalization of this model, the LOD score contribution of a single pair is

$$
\log_{10}\left\{\frac{\hat{f}_0+\hat{f}_1e^{\beta_1}+\hat{f}_2[(\alpha+1)e^{\beta_1}-\alpha]}{f_{r0}+f_{r1}e^{\beta_1}+f_{r2}[(\alpha+1)e^{\beta_1}-\alpha]}\right\},\,
$$

where $\alpha \geq 1$ is a mode of inheritance parameter: $\alpha = 1$ (corresponding to Whittemore & Tu's $w_1 = a = 0.5$) gives a dominant model and $\alpha \rightarrow \infty$ (corresponding to Whittemore & Tu's $w_1 = a = 0$) gives a recessive model. (In practice, $\alpha \approx 10$ gives a pretty good recessive model.) Compared to the two-parameter model, this model has the constraints $\lambda_2 = (\alpha + 1) \lambda_1 - \alpha$ in terms of relative recurrence risks and $\beta_1 \geq 0$ (default).

[1](#page-196-0)1.2.2.3 $Covariates¹$

Inclusion of a single covariate (*z*) gives

¹Covariates are pair-specific and are allowed only in the one-parameter model.

$$
\log_{10}\left\{\frac{\hat{f}_0+\hat{f}_1e^{\beta_1+z\delta}+\hat{f}_2[(\alpha+1)e^{\beta_1+z\delta}-\alpha]}{f_{r0}+f_{r1}e^{\beta_1+z\delta}+f_{r2}[(\alpha+1)e^{\beta_1+z\delta}-\alpha]}\right\},\right
$$

where δ is an additional parameter to be estimated and *z* is the adjusted (see below) value of the covariate for that pair. The model extends easily to include more than one covariate; there is one parameter δ_c for each covariate.

• Constraints on the δ_c :

Let the original (unadjusted) covariate value be denoted *x*. Two options are allowed:

- 1. Genetic constraints on β_1 hold at the average value, $x = \bar{x}$, but not necessarily for all *x*. The covariate value is centered to give $z = x - \bar{x}$ before inclusion in the likelihood, so that the mean of the centered covariate = 0. Then δ_c is unconstrained.
- 2. Genetic constraints on β_1 hold at all values of *x*. The minimum value of a covariate is subtracted (i.e., $z = x - \min(x)$), so that the smallest value of the covariate equals zero. Then, for a set of covariates indexed by *c*, the following constraint is applied:

$$
\min_{z>0}\sum_{c}z_c\delta_c\geq -\beta_1.
$$

11.2.3 Adding Discordant Sib Pairs (DSPs) to an ARP Analysis (one-parameter model only)

This model is the same as the ARP one parameter model with one covariate that indicates nonconcordance status:

$$
\lambda_1=e^{\beta_1+z\delta},
$$

where the covariate ζ is set to

- 0 if the pair is concordant and
- 1 if the pair is discordant for affection status.

A related model sets the covariate *z* to

- 0 if the pair is concordantly affected (concordantly unaffected pairs are not used in the analysis) and to
- 1 if the pair is discordant for affection status.

When either option is chosen, β_1 and δ are estimated subject to the constraints: $\beta_1 \geq 0$, $\delta \leq -\beta_1$.

No additional covariates may be included when discordant sib pairs are included in the analysis this way.

11.2.4 Contrasting Discordant Relative Pairs (DRPs) to Affected Relative Pairs (ARPs)

This model is the same as the ARP one- or two- parameter model except that the prior probabilities of sharing 0, 1, or 2 alleles IBD given a relative pair of type $r(f_{r0}, f_{r1}, f_{r2})$, are replaced by the probabilities, given the data, of sharing 0, 1, or 2 alleles IBD by the corresponding discordant relative pairs (Shih et. al. (2005)). With this option, any pair type for which there are no contrasting discordant relative pairs in the data is not included in the analysis. Additional covariates can be included in the case of a one-parameter model.

11.2.5 X-linked Models

Models for X-linkage are similar to those for autosomal inheritance. Recall the autosomal model: the LOD score contribution for a particular affected relative pair (ARP) of type *r* is

$$
\log_{10}\left\{\frac{\sum\limits_{i=0,1,2}\hat{f}_i\lambda_i}{\sum\limits_{i=0,1,2}f_{ri}\lambda_i}\right\}.
$$

For X-linked models, the LOD score is

$$
\log_{10}\left\{\frac{\sum\limits_{i=0,1,2}\hat{f}_{iuv}\lambda_{iuv}}{\sum\limits_{i=0,1,2}fr_{iuv}\lambda_{iuv}}\right\},\right.
$$

where u , v denote the sex ($m =$ male, $f =$ female) of the members of the pair. For male-female ARPs, *m* and *f* are interchangeable, i.e. $\lambda_{imf} = \lambda_{ifm}$.

There are four possible relative risk parameters: λ_{1ff} , λ_{2ff} , λ_{1mm} , and λ_{1mf} (= λ_{1fm}). All others equal 1 (e.g., $\lambda_{0ff} = \lambda_{2mm} = 1$, etc.). The following table gives the λ parameters for each type of ARP.

• Constraints on λ_{1ff} , λ_{1mm} , λ_{1mf} :

– DEFAULT VALUE : all λ_1 constrained to be equal:

$$
\lambda_{1ff}=\lambda_{1mm}=\lambda_{1mf}
$$

- OPTIONAL : all λ_1 not constrained to be equal: λ_{1ff} , λ_{1mm} , and λ_{1mf} are estimated separately.
- Constraints on λ_{2ff} :
- $-$ DEFAULT VALUE : $λ_{2ff} = (α+1)λ_{1ff} α$ The default value of α is 2.634.
- OPTIONAL : λ_{2ff} is not constrained to be dependent on λ_{1ff} . λ_{1ff} , and λ_{2ff} are estimated separately.

Since both parameters are not separately estimable if the data contain only ASPs or no ASPs, unless λ_{1ff} is estimated in part using male-male and/or male-female ASPs, this option will be carried out only if either the data contain at least 15 male-male and malefemale sib pairs (ASPs) under the default constraints on λ_1 , or if the data contain at least 15 sister-sister ASPs and at least 15 female-female ARPs other than ASPs under the optional constraints on λ_1 .

Under this model, the additional constraint $\beta_{2ff} \ge \log_e(2e^{\beta_{1ff}} - 1)$ $\beta_{2ff} \ge \log_e(2e^{\beta_{1ff}} - 1)$ $\beta_{2ff} \ge \log_e(2e^{\beta_{1ff}} - 1)$ is used².

11.2.5.1 Covariates

Inclusion of a single covariate (*z*) gives

$$
\log_{10}\left\{\frac{\widehat{f}_{0uv}+\widehat{f}_{1uv}e^{\beta_{1uv}+z\delta_{uv}}+\widehat{f}_{2uv}[(\alpha+1)e^{\beta_{1uv}+z\delta_{uv}}-\alpha]}{f_{r0uv}+f_{r1uv}e^{\beta_{1uv}+z\delta_{uv}}+f_{r2uv}[(\alpha+1)e^{\beta_{1uv}+z\delta_{uv}}-\alpha]}\right\},
$$

where, δ_{uv} is an additional parameter to be estimated and *z* is the adjusted value of the covariate for that pair as with the autosomal models. The model extends easily to include more that one covariate; there is one additional parameter for each covariate.

Under a generalization of this model, the LOD score is

$$
\log_{10}\left\{\frac{\hat{f}_{0uv}+\hat{f}_{1uv}e^{\beta_{1uv}+\sum_{c}z_{c}\delta_{uv}}+\hat{f}_{2uv}[(\alpha+1)e^{\beta_{1uv}+\sum_{c}z_{c}\delta_{uv}}-\alpha]}{f_{r0uv}+f_{r1uv}e^{\beta_{1uv}+\sum_{c} \delta_{uv}z_{l}}+f_{r2uv}[(\alpha+1)e^{\beta_{1uv}+\sum_{c}z_{c}\delta_{uv}}-\alpha]}\right\},\right.
$$

where c indexes the covariate. Note that covariates can only be included in the one-parameter model, with the constraint $\lambda_{2ff} = (\alpha+1)\lambda_{1ff} - \alpha$.

Constraints on the δs are the same as for the autosomal models.

11.2.6 Parent-of-Origin Models

The expression of an allele may depend on the sex of the parent from whom the allele was inherited; this phenomenon is known as a parent-of-origin effect (alternatively, genetic imprinting). For example, individuals affected with the autosomal dominant condition Beckwith-Wiedemann syndrome almost always inherited the defective allele from their mother. Individuals who inherit the defective allele from their father are rarely affected with this disorder.

For the model that includes a parent-of-origin effect, the ARP lod score model fits separate parameters for the maternal and paternal effects. The test of parent-of-origin effect is obtained by

²As with the autosomal models, β in $\lambda_{iuv} = \exp(\beta_{iuv})$ is estimated instead of estimating λ itself.

comparing the likelihood-ratio statistics (i.e., 4.6 times the lod score) for the models with and without the parent-of-origin effect. The parent-of-origin model can only be applied to autosomal loci.

For the parent-of-origin model, the LOD score for a particular affected relative pair is

$$
\log_{10}\left\{\frac{\sum\limits_{i=0,1m,1p,2}\hat{f}_i\lambda}{\sum\limits_{i=0,1m,1p,2}f_{ri}\lambda_i}\right\},\right
$$

where *m* denotes maternal and *p* denotes paternal, so that the sum is over $i = 0$, 1*m*, 1*p*, 2 rather than over $i = 0, 1, 2$. As in previous models, $\lambda_0 = 1$ and $\lambda_i = \exp(\beta_i)$, where β_i is the parameter estimated.

11.2.6.1 One Parameter Model

First note that $\lambda_1 = \frac{\lambda_{1m} + \lambda_{1p}}{2}$ $\frac{1+\lambda_1 p}{2}$. The one-parameter model employs the same mode-of-inheritance constraint, i.e., $\lambda_2 = (\alpha + 1) \lambda_1 - \alpha$.

11.2.6.2 Covariates

Covariates may be included only in the one-parameter model. Inclusion of a single covariate (*z*) gives $\lambda_{1m} = e^{\beta_{1m} + z\delta_m}$ and $\lambda_{1p} = e^{\beta_{1p} + z\delta_p}$, where δ_m and δ_p are the additional parameters to be estimated and *z* is the adjusted value of the covariate for that pair, as with the autosomal models. The model extends easily to include more that one covariate; there are two additional parameters for each covariate, so that $\lambda_{1m} = e^{\beta_{1m} + \sum_c z_c \delta_m}$ and $\lambda_{1p} = e^{\beta_{1p} + \sum_c z_c \delta_{lp}}$, where *c* indexes the covariate under the generalization of this model. We include an option that fixes either λ_{1m} or λ_{2m} to be equal to 1. In such situations, only one covariate parameter is fitted for each covariate.

Constraints on the δs are the same as with the other autosomal models.

These models only apply to ASPs and affected half-sib pairs because the problem of computing the right IBD probabilities for other types of ARPs is daunting. By default, other types of ARPs are excluded from the analysis, but an option to include other types into the analysis is provided. When other types of ARPs are included in an analysis with a parent-of-origin effect, λ_1 is replaced with $\{(\lambda_{1m} + \lambda_{1p})/2\}$ in the other ARPs to avoid fitting an extra parameter. The only information about parent-of-origin effect in the models comes from the ASPs, and so parent-of-origin models will not be allowed if the number of ASPs in which $\hat{f}_{1m} \neq \hat{f}_{1p}$ is less than 10, even if many other ARPs are available. (It should be recognized that, for other types of ARPs, parent-of-origin effects may be highly confounded with ascertainment.)

11.2.7 Asymptotic P-value and Empirical P-value

Two p-values, asymptotic and empirical, are reported under the conditions listed below:

- Autosomal locations
- One-parameter model
- Affected pairs only
- Pair count is between 20 and 350
- Number of covariate is less than 5
- Average location distance is between 1cM and 20 cM in case using multipoint ibd information.

Asymptotic p-values are computed using 50:50 mixture of χ^2 with *c* and $c + 1$ degrees of freedom for *c* covariates. Empirical p-values are computed using the results described in Sinha et. al. (2006). Note that no p-values are computed when any of these conditions are not met!

11.3 Program Input

Note:

To use the X-linked model in LODPAL, an IBD sharing file has to include " x_linked" after the name of the marker in the file header.

11.3.1 Running lodpal

A typical run of the LODPAL program may use flags to identify the file types like the following:

>lodpal -p data.par -d data.ped -i ch1.ibd

or, rely on a set file order like the following:

>lodpal data.par data.ped ch1.ibd

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file, and ch1.ibd is the name of the IBD sharing file.

11.3.2 The lodpal Block

A lodpal block in the parameter file sets the options on how to perform an analysis using LODPAL. The following table shows the syntax for a lodpal parameter which starts the lodpal block.

The following table lists the parameters and attributes that may occur in a lodpal block.

Notes

- 1. The value of a trait parameter should be set to the name of a binary trait or covariate field read from the data file or created by means of a function block.
	- (a) If no valid trait parameters are listed, then all trait fields read in from the data file are used.
	- (b) If more than one trait is specified, then each will be used in a separate analysis.
	- (c) If a trait is not a binary trait, then it will be dichotomized at 0 (trait values $<= 0$ will be treated as unaffected and values > 0 will be treated as affected) or at the value of the cutpoint attribute. When dichotomizing a trait using a cutpoint, all values less than or equal to the cutpoint are considered unaffected and all values strictly greater than the cutpoint are considered to be affected.
	- (d) If no attributes are listed, then by default the conaff attribute is assumed. This attribute causes the program to select only concordantly affected relative pairs (ARPs) and perform an analysis on these pairs.
	- (e) If a trait parameter has the condisc attribute, then the program pools the concordantly affected relative pairs with the concordantly unaffected sib pairs and performs a one-parameter model analysis in which these are analyzed together with the discordant sib pairs (DSP) by creating a covariate to indicate concordance status of the pairs.
	- (f) If a trait parameter has the noconunaff attribute, the program performs the same analysis as with the condisc attribute, but without including the concordantly unaffected sib pairs.
	- (g) When either the condisc attribute or the noconunaff attribute is used, no covariates can be included. If the user specifies any covariates, they are ignored by the program.
	- (h) If a trait parameter has the contrast attribute specified then the program computes the probabilities of sharing 0, 1 or 2 alleles IBD for a given relative pair type from the discordant relative pairs in the data, and the prior probabilities are replaced by these values. Additional covariates can be included in the case of a one-parameter model.
- 2. The trait specified by a subset parameter should be a binary trait coded as 0 for individuals to be excluded from, and 1 for individuals to be included in, the analysis. The subset parameter may be included more than once, in which case the only individuals included in the analysis are those for which all the indicated binary traits are coded 1.
- 3. The value of a marker parameter should be set to the name of a marker (or marker location) for which IBD sharing information was generated and stored in the IBD sharing file. The marker parameter may be included more than once. If no valid marker parameters are listed, then all markers are used.
- 4. The value of a covariate parameter should be set to the name of a trait or covariate field read from the data file or created by means of a function block. The covariate parameter may be included more than once. A covariate parameter may have two attributes, separated by a comma, to specify the function to compute a pair-specific value from two individual-specific values (sum, diff, single, avg or prod), and to adjust the covariate value to impose genetic constraints on them (mean or minimum).
- (a) Individual covariate values are not mean-corrected, neither in LODPAL nor in SIBPAL. In the SIBPAL program, pair-specific covariate values are mean-corrected as stated in the manual. In LODPAL, however, pair-specific covariate values are either meancorrected (by default) or minimum-adjusted (if minimum attribute is specified) as stated also in the manual.
- (b) If sum, diff, avg or prod attributes are specified, then a single covariate sum, absolute difference, average or product term of two individual-specific values is included as a pair-specific covariate value.
- (c) If the both attribute is specified, then both sum and difference terms are included.
- (d) If the single attribute is specified, then the covariate value for the first member of the pair is included as a pair-specific value.
- (e) If no attribute for the covariate parameter is specified, then the sum attribute is applied by default.
- (f) If the mean attribute is specified, then the program automatically centers each pairspecific covariate value before inclusion in the likelihood, using the sample mean or a user-supplied value (for example, mean $= 0.5$).
- (g) If the minimum attribute is included, then the program automatically puts the offset from the smallest observed covariate value as the pair-specific covariate value into the likelihood, so that the smallest value of the pair-specific covariate equals zero.
- 5. The value of a diagnostic parameter should be set to the name of a marker or a location (in centiMorgans) for which IBD sharing information was generated and stored in the IBD sharing file. If the diagnostic parameter has a valid value, then an additional output file, "lodpal.lod", will be generated that contains the individual pair LOD score contributions for the final model at the particular location specified by the diagnostic parameter value.
- 6. If the program finds the turn_off_default parameter , then the program maximizes the LOD score in a somewhat simpler way than the default way. By default, the program uses a method that avoids, as much as possible, spuriously high LOD scores. However, because there may be multiple true maxima, the result obtained using the turn_off_default parameter may also be of interest.
- 7. If the wide_out parameter is set to true, then additional columns are added to the output of the LOD score Analysis of Affected Relative Pairs. The information contained in the additional columns are :
	- detailed information on the number of pairs
	- partial first derivative of the maximum LOD score with respect to each of the parameters
	- the number of iterations it took for maximization
- 8. If the program finds the pair info file parameter with a valid file name, then the program uses the pre-constructed pair-specific covariate values from the file specified. The pair_info_file parameter may have its own sub-block to specify the name of the pairspecific covariate(s) to be used in the current analysis.
- 9. If the program finds the autosomal or autosomal_model parameter, then the program uses the specified autosomal model for the autosomal locations. The autosomal parameter may

have its own sub-block to specify the model to be used in the current analysis. If no subblock is found, the default autosomal model will be used, i.e., the one-parameter model with the default alpha value, and without parent-of-origin effect.

10. If the program finds the x_linkage or x_linkage_model parameter, then the program uses an X-linked model for the X-linked markers. The x -linkage parameter may have its own sub-block to specify the model to be used in the current analysis. If no sub-block is found, the default, the X-linked model will be used, in which all three λ_1 parameters are constrained to be equal, and λ_{2ff} is fixed.

11.3.2.1 The pair_info_file Sub-Block

The following table lists the parameters and attributes that may occur in a pair_info_file subblock.

Notes

- 1. The value of a pair_covariate parameter should be set to the name of a covariate field read from the Pair Information File. The pair_covariate parameter may be included more than once.
- 2. The pair_covariate parameter may be included more than once. A pair_covariate parameter may have an attribute to adjust the covariate value to impose genetic constraints on them (mean or minimum).
- (a) If the mean attribute is specified (the default), then the program automatically centers each pair-specific covariate value before inclusion in the likelihood, using the sample mean or a user-supplied value (for example, mean $= 0.5$).
- (b) If the minimum attribute is included, then the program automatically puts the offset from the smallest observed covariate value as the pair-specific covariate value into the likelihood, so that the smallest value of the pair-specific covariate equals zero.

11.3.2.2 The autosomal Sub-Block

The following table lists the parameters and attributes that may occur in an autosomal sub-block.

Notes

- 1. The value **one_parameter** specifies the Whittemore and Tu one-parameter model. The value two_parameter specifies the two-parameter model. The two-parameter model does not allow the inclusion of covariate data. If **two_parameter** is specified, any covariate parameter is ignored and no covariates are included in the analysis.
- 2. If this parameter is not specified, the Holmans triangle constraints are applied.
- 3. A fixed value of **maternal** sets λ_{1m} equal to 1, and a fixed value of **paternal** sets λ_{1p} equal to 1.
- 4. By default, other types of affected relative pairs are excluded from the analysis to reduce the computational complexity of calculating IBD sharing probabilities for other types of affected relative pairs. If the all_pairs attribute is specified, then other types of affected relative pairs are included in the analysis, but the parent-of-origin effect test is applied only to the affected sib pairs. The other types of affected relative pairs are included with λ_1 be replaced by $\{(\lambda_{1m} + \lambda_{1p})/2\}$ to avoid fitting an extra parameter.

11.3.2.3 The x_linkage Sub-Block

The following table lists the parameters and attributes that may occur in a x_linkage sub-block.

The following are all valid LODPAL statements:

```
lodpal
{
   \text{trait} = \text{T1}}
lodpal
\left\{ \right.\text{trait} = T1
   marker = M1
  covariate = ageexam, minimum, diff
}
lodpal
{
   trait = T1
  autosomal_model
   \mathbf{f}model = two_parameter
   }
  diagnostic = "20 44.0" # The additional output file for location "20 44.0" is generated.
}
lodpal, out="t1condisc.out"
{
  trait = T1,condisc
   turn_off_default # Turn off default maximization process
}
lodpal
\left\{ \right.trait = T1, noconunaff \qquad # Analysis is done with the one-parameter model.
}
lodpal
{
  trait = T1
  pair_info_file = "cov.in"
   {
      pair_weight = probability
     pair_covariate = covariate1, mean
  }
}
lodpal
{
   trait = T1
   x_linkage
   {
      lambda2_fixed = true, alpha = 3.5 # The same as default model,
                                          # but the different alpha value is used.
```

```
}
}
lodpal
{
   \text{trait} = \text{T1}x_linkage
   {
      pair_type = "M-M"lambda1_equal = false # All three lambdas are estimated separately.
      lambda2_fixed = false # The data set has to have at least 15 sister-sister pairs
                              # and at least 15 female-female pairs other then
                              # sister-sister pairs to use this model.
  }
}
```
11.3.3 Pair Information File

The pair information file is a character delimited file that stores the pre-constructed pair-specific covariate values for the pairs to be used in the analysis. The first line of the file is the header that contains the name of each field, and the rest of the file contains one line for each pair, with the required IDs and covariate fields. The pedigree ID (PEDID in the example below), first individual ID (ID1 in the example), and second individual ID (ID2 in the example) fields are required in that order, and covariate fields can be in any order. Each individual is expected to be found in the data file, and the pairs are expected to be found in the IBD sharing file, for the analysis to proceed. Any individual or pair that is not in both of these files will be ignored. The weight and covariate values should be numerics, and no missing values are allowed.

A pair information file may look like the following:

Another Pair Information File may look like:

PEDID, ID1, ID2, weight1, covariate1 1,3,4,0.0033619,0.0033619 102,3,6,0.0114638,0.0000000 102,6,7,0.0022620,0.3283151 102,3,7,0.0162358,0.0000000 104,5,6,0.9802018,0.0000000 105,6,7,0.0135131,0.9079691 106,3,4,0.8125513,0.0334500 107,7,8,0.9497964,0.0006405 . . .

11.4 Program Output

LODPAL produces several output files that contain results and diagnostic information:

11.4.1 Pair Analysis Output File

One pair analysis output file, named either "LODPAL.out" or "LODPAL.xln", is generated per run of LODPAL. It contains tables of LOD scores and parameters estimates for each marker location tested.

Example:

```
==============================================================
 Conditional Logistic Analysis of
   Affected Relative Pairs - multipoint
==============================================================
  Trait : affection
              concordantly affected relative pairs
   Covariate: cov1
              sum of two individual covariate values
              mean centered
              mean before adjusting = 0.232673
              mean after adjusting = 0.000000std. deviation = 0.423585Method : default analysis method
   Model : one-parameter model, constrained, alpha = 2.634
==============================================================
                        Full Parameter Estimates<br>Sib All -------------------
                LOD Sib All -------------------
MARKER cM SCORE Pairs Pairs Beta1 cov1
----------------- --------- ------ ------ --------- ---------
20s103 ------ 0.080913 117 202 0.053795 -0.053795
 20_2.0 2.0 0.064902 117 202 0.051271 -0.051271
 20_4.0 4.0 0.048328 117 202 0.045973 -0.045973
 20_6.0 6.0 0.033579 117 202 0.038439 -0.038439
20_8.0 8.0 0.022750 117 202 0.030557 -0.030557
 20s482 ------ 0.019024 117 202 0.027139 -0.027139
 20_10.0 10.0 0.020959 117 202 0.029545 -0.029545
 20_12.0 12.0 0.025838 117 202 0.035060 -0.035060
20_14.0 14.0 0.031819 117 202 0.041004 -0.041004
.
.
.
==============================================================
```
11.4.2 Diagnostic Output File

One diagnostic output file, named "LODPAL.lod" by default, is generated per run of LODPAL when a valid diagnostic location (i.e., marker) has been specified by the user. It contains a table of individual LOD score contributions, covariates, and allele-sharing probabilities at the specified location, along with a variance-covariance matrix of the parameter estimates (assuming independence of all pairs) and a histogram of the individual LOD score contributions.

Example:

```
===================================================================================
 Conditional Logistic Analysis of Affected Relative Pairs - multipoint
===================================================================================
   Trait : affection
              concordantly affected relative pairs
   Covariate: cov1
               sum of two individual covariate values
              mean centered
              mean before adjusting = 0.232673
              mean after adjusting = 0.000000<br>std. deviation = 0.423585std. deviation
   Method : default analysis method
   Model : one-parameter model, constrained, alpha = 2.634
   Location : 20_36.0
  ===================================================================================
# Final Result Summary
   Parameter Estimates:
     1. beta 1 = 0.121141
     2. cov1(delta 1) = 0.132373
   Variance-Covariance Matrix(assuming independent pairs):
      ------------------------------------
     | \ | 1 | 2 |
     ------------------------------------
     | 1 | 0.017046 | 0.003377 |
              ------------------------------------
     | 2 | 0.003377 | 0.001245 |
        ------------------------------------
# Histogram of Individual LOD Score Contributions
   Maximum LOD Score = 0.5936
   Minimum LOD Score = -0.4504
   Bin Size = 0.1044
                    Count (one * is equal up to 2 pair(s).)
-----------------------------------------------------------------------------------
-0.4505 to -0.3461 2 *<br>-0.3461 to -0.2417 4 **
-0.3461 to -0.2417 4 **<br>-0.2417 to -0.1372 7 ****
-0.2417 to -0.1372-0.1372 to -0.0328 47 ************************
-0.0328 to 0.0716 89 *********************************************
 0.0716 to 0.1760 35 ******************
 0.1760 to 0.2805 13 *******
0.2805 to 0.3849 4 **
0.3849 to 0.4893 0
0.4893 to 0.5937 1 *-----------------------------------------------------------------------------------
             Total : 202
# Individual LOD Score Contribution
FAMID IDSIB1 IDSIB2 F0 F2 Cov. prin2 LOD SCORE CONTRIBUTION
-------- -------- -------- ---------- ---------- --------------- ---------------
1 3 4 0.0033619 0.0033619 3.5349345 -0.0715001694
102 3 6 0.0114638 0.0000000 0.9441527 0.0490763128
109 3 4 0.0028969 0.1407563 -1.0821854 0.0003981387
-------- -------- -------- ---------- ---------- --------------- ---------------
                                               Total LOD Score = 3.3944467415===================================================================================
```
Chapter 12

MARKERINFO

MARKERINFO detects Mendelian inconsistencies in pedigree data. Each marker is individually checked for inconsistencies in every constituent pedigree. These inconsistencies are sorted by marker, by pedigree, and by whether one or more than one nuclear family is involved in the inconsistency.

12.1 Limitations

MARKERINFO assumes codominant markers, analyses one marker at a time and is only guaranteed to detect all errors in the absence of loops. Mendelian inconsistencies cannot be localized beyond the nuclear family in which they are first detected (see theory).

12.2 Theory

The phenoset of an individual is the set of all genotypes consistent with that individual's marker phenotype. Individuals labeled as missing are considered to be consistent with all possible marker phenotypes. MARKERINFO detects Mendelian inconsistencies in pedigree data by reducing the set of possible genotypes for each individual to the minimal possible subset on the basis of both the individual's phenoset and the phenosets of surrounding individuals. An empty minimal subset of genotypes for any individual indicates a Mendelian inconsistency.

Example 1

In this example pedigree, individuals 1 and 3 have a phenoset consisting of genotype A/A, while individual 4 has a phenoset consisting of genotype B/B. Individual 2 is unknown, so her phenoset includes all possible genotypes: {A/A, A/B, B/B, etc.}

These phenosets are reduced based on Mendelian inheritance from parent to child. Under Mendelian inheritance, a parent having marker genotype A/B can transmit either the A or the B allele to the child, but cannot transmit any other allele at that marker. Any genotype for which there is no valid transmission from a parent or to a child is removed from the phenoset. In this way, the subset of possible genotypes for individual 2 becomes A/B and that for individual 1 become empty.

MARKERINFO detects two sorts of inconsistencies, those involving one, and those involving more than one, nuclear family. In the above example, there is no valid transmission from individual 1 to individual 4 because 4 must receive a B allele from both parents and 1 has no B allele. In this and the next example it is sufficient to inspect a single nuclear family to detect an inconsistency.

Example 2:

In this example, each of the children must receive a different set of alleles from each of their parents, but each parent has only two alleles. At least one child must be inconsistent with the parents, but it is impossible to determine which one.

Inconsistencies Involving More than one Nuclear Family

Often, a single nuclear family appears consistent until new information is added from surrounding nuclear families. Consider example 3.

Example 3:

Looking at only the nuclear family with parents 1 and 2, we see that this family is consistent, with 1 and 2 each having subset of possible genotypes A/C, B/D. Note here that if 1 is A/C, 2 must be B/D and vice versa. From this, we can deduce that the subset of possible genotypes for 6 is A/D, A/B, C/D, B/C.

Similarly, the nuclear family with parents 6 and 7 is consistent, with the subset of possible genotypes for 6 being A/C. However, A/C is not present in the subset of possible genotypes for 6 as derived from the first nuclear family. There is no genotype present in both subsets, so the minimal subset is empty. Because the sequence in which MARKERINFO traverses the pedigree depends on several factors, the inconsistency could be first detected in either of the nuclear families, and only one of them will be reported as being inconsistent.

12.3 Program Input

12.3.1 Running markerinfo

A typical run of the MARKERINFO program may use flags to identify the file types like the following:

>markerinfo -p data.par -d data.ped

or, rely on a set file order like the following:

>markerinfo data.par data.ped

where data.par is the name of the parameter file and data.ped is the name of the pedigree data file.

12.3.2 The markerinfo Block

A markerinfo block in the parameter file sets the options on how to perform an analysis using MARKERINFO.

The following table shows the syntax for a markerinfo parameter which starts the markerinfo block.

The following table lists the parameters and attributes that may occur in a markerinfo block.

Notes

- 1. The value of sample_id should be set equal to the name of a string field read from the pedigree data file. This can be used to indicate the location where a sample is stored.
- 2. If consistent_out is set to true, then the nuclear family members who are not inconsistent are added to the output with [] around them.
- 3. If pedigree_out is set to true, a new pedigree file, along with a corresponding new parameter file, is produced in which, for all members of the pedigree with any inconsistency, those inconsistent markers are set to missing.

12.4 Program Output

SIBPAL produces several output files that contain results and diagnostic information:

Note:

Two types of Mendelian inconsistencies are differentiated: those which occur within a single nuclear family, and those in which members of more than one nuclear family are involved – i.e. the inconsistency can only be detected if two or more nuclear families are simultaneously examined. In the latter case, only one of the nuclear families that could be involved is shown in the output, followed by *.

12.4.1 Analysis Output File

Here is a typical example of MARKERINFO output:

221

.

Chapter 13

MLOD

MLOD Performs multi-point model-based LOD-score linkage analysis on small constituent pedigrees. Analysis is optimized for examining one-locus trait models across the genome.

13.1 Limitations

MLOD calculates the likelihood of each possible inheritance pattern (i.e., ancestral origin of each allele) at each marker location for each constituent pedigree, using all marker data and assuming no crossover interference. It is restricted to small pedigrees due to the exponential nature of the algorithm related to the number of individuals in the pedigree. Only discrete traits may be analyzed, but there is no limit on the number of discrete categories allowed (this effectively allows the analysis of quantitative traits). The time and space complexity of the algorithm is largely characterized by the number of genomic locations examined and the exponent $2n - f$, the number of bits in an inheritance vector, where n is the number of non-founders and f is the number of founders in a constituent pedigree. During parameter specification the maximum value of 2*n*− *f* may be set, so that any constituent pedigree that has a value larger than this maximum will be skipped. X-linked markers cannot be analyzed.

13.2 Theory

Given trait-marker (see 3.3, 3.2.5.4), *T*, and marker data, M, for a chromosomal region, and a point of interest in that region, *p*, MLOD computes a multi-point LOD score, defined as:

$$
Z(p) = \log_{10}\left(\frac{P(M | T at p)}{P(M)P(T)}\right),\,
$$

where $P(T)$ can be a probability mass or density function.

Given a chromosomal region, a trait, and several pedigrees, MLOD calculates multi-point LOD scores for each location of interest along the chromosome by first generating exact multi-point likelihoods at each marker location using a modified Lander-Green approach (Idury and Elston, 1996), and then computing the likelihood for the trait of each inheritance pattern (which is proportional to the probability of the trait for each inheritance pattern). These likelihoods are combined to generate the final LOD score at each location specified by the user.

13.2.1 The Exact Multi-point Algorithm

The general algorithm used by MLOD to generate multi-point likelihoods and other related statistics is called the exact multi-point algorithm. This algorithm takes a chromosomal region and generates likelihoods of all the possible inheritance patterns at each marker location in the region. These likelihoods are then combined at each marker location to generate multi-point LOD scores.

Given a pedigree with *f* founders and *n* non-founders and a pattern of segregation at a particular locus for this pedigree, we may represent this segregation as a vector of binary (0 or 1) digits of length 2*n* where each element represents one of the 2*n* meioses in the pedigree. The value of each binary element is determined by that meiosis receiving either a grandpaternal or grandmaternal allele from the parent. This "inheritance vector" is the basis for the Lander-Green multi-point algorithm (Lander and Green, 1987).

Because each meiosis is a separate event at a given locus, there are 2^{2n} possible patterns of locus segregation in the pedigree for each marker. However, because founder phase is unknown, it is impossible to determine the true state of the meioses from the founders. This means that, for the founder meioses, we do not know the binary values to be used in the inheritance vectors for a given inheritance pattern. Each inheritance pattern can therefore be represented by 2^f different inheritance vectors that represent the same inheritance pattern and share the same likelihood. These "equivalence classes" of inheritance vectors reduce the number of vectors that we must consider to 2^{2n-f} .

For a given set *M* of *i* markers $m_1 \ldots m_i$ (including a trait-marker, i.e. a trait considered in the same manner as any other marker but with more general penetrance functions), we calculate the joint probability of each inheritance vector and the pedigree data at each marker. The set of 2^{2*n*−*f*}joint probabilities at a particular marker is called the *likelihood vector* for that marker. The sum of these 2^{2*n−f*} joint probabilities is proportional to the likelihood for the pedigree data.

13.2.2 Combining Likelihood Vector Elements to Obtain a Multi-point Likelihood

Given two likelihood vectors, v_1 and v_2 at markers m_1 and m_2 , and a recombination fraction θ_1 between them, we wish to calculate the joint likelihood.

To do this, we form a transition matrix T_1 . This is a $2^{2n-f} \times 2^{2n-f}$ matrix with elements $t_{\alpha\beta}$ = θ_1^q $\int_1^q (1-\theta_1)^{2n-f-q}$ where α , β are inheritance vectors of the two markers and *q* is the Hamming Distance between them (the number of elements of α , β that differ). Then,

$$
L(v_1, v_2) = v'_1 T_1 v_2.
$$

To add a third likelihood vector v_3 at marker m_3 , with recombination fraction θ_2 between m_2 and m_3 , we form a transition matrix T_2 analogous to T_1 . Then

 $L(v_1, v_2, v_3) = v'_1T_1V_2T_2v_3$, where V_2 is a $2^{2n-f} \times 2^{2n-f}$ diagonal matrix containing the elements of *v*2.

In general,

$$
L(v_1, v_2, \ldots v_{i-1}, v_i) = v'_1 T_1 V_2 T_2 \ldots V_{i-1} T_{i-1} v_i.
$$

Idury and Elston (1996) suggested methods of calculating these likelihoods that are efficient, given the underlying structure of the transition matrices. S.A.G.E. extends these methods to include additional optimizations that use the genetic information at the markers to reduce the time complexity

of these algorithms. Even so, the algorithm takes time and space that increases exponentially with the size of the pedigree. It is for this reason that these algorithms are restricted to small-to-medium sized pedigrees.

13.2.3 Using Genetic Information to Improve Algorithm Performance

There are 2^{2n-f} inheritance vectors that we must consider at each marker. However, when most individuals are typed, the joint probability of the data and many of these inheritance patterns will be zero, because the inheritance pattern indicated by the vector is not consistent with the observed phenotypes at the marker in question.

A *fixed point* is any meiosis where the transmission is known with certainty. Given a fixed point in our likelihood vector, all inheritance vectors that do not match the transmission of the fixed point have a joint probability of 0. This information is used to speed up the computation. For each fixed point, we can reduce the time required for calculation by a factor of 2. These reductions are cumulative, so that for *n* fixed points, the time is reduced by a factor of 2^n .

13.2.4 Calculating Multi-point Likelihood Vectors

It is often necessary to calculate the multi-point likelihood vector at a specific location *p* along a chromosome. Assume we have a chromosome containing markers m_1, \ldots, m_i with distances d_1 ,..., d_{i-1} between them. We have two adjacent markers, m_i and m_{i+1} between which is a point *p* for which we wish to calculate a multi-point likelihood vector v , with p some known distances d_{i1} and d_{j2} (where $d_{j1} + d_{j2} = d_j$) from m_j and m_{j+1} , respectively. Distances are expressed as recombination fractions and may are translated from genetic distance using either the Kosambi or Haldane map function.

First, we calculate v_1, \ldots, v_i , the single-point likelihood vectors for each marker. Then we calculate the following:

$$
P_{j1} = v'_1 T_1 V_2 T_2 \dots V_j \text{ and } P_{j2} = v'_i T_{i-1} V_{i-1} \dots V_{j+1}.
$$

 P_{j1} is the multi-point information contributed to point *p* by all markers before point *p*, while P_{j2} is the multi-point information contributed by all markers after *p*. Each is a $1 \times 2^{2n-f}$ vector representing the combined multi-point information contributing to ν . Calculating ν is now trivial:

$$
v = P_{j1}T_{j1}T_{j2}P'_{j2}
$$

where P'_{j2} is a diagonal matrix consisting of elements of P_{j2} .

13.2.5 Computing LOD Scores

For a given point *p* on a chromosome, we calculate the multi-point LOD score given that a trait locus *T* (the trait-marker), is at that location by first calculating $P(M|T \text{ at } p)$, the multi-point likelihood for the chromosome given that *T* is present at that location and follows the model specified. We then calculate, $P(M)$, the multi-point likelihood for the chromosomal region without *T*, and $P(T)$, the probability of the trait given the underlying model. Then the LOD score for *T* being at point *p* is

$$
Z(p) = \log_{10}\left(\frac{P(M | T at p)}{P(M)P(T)}\right).
$$

At each location *p* we generate a LOD score for each pedigree. The combined LOD score at *p* is the sum of each constituent pedigree's individual LOD score at *p* .

13.2.6 Computing Information Content

Information content at a location is determined based on the probabilities of each inheritance pattern within the likelihood vector at that location. If we have *n* possible inheritance patterns, $i_1 \ldots i_n$, each with *b* bits and probability p_i such that

$$
\sum_i p_i = 1,
$$

then, the Information *I* is defined by [Kruglyak and Lander, 1995b]

$$
I = 1 + \frac{\sum_{i} p_i \frac{\log(p_i)}{\log(2)}}{b}.
$$

13.3 Program Input

13.3.1 Running mlod

A typical run of the MLOD program may use flags to identify the file types like the following:

>mlod -p data.par -d data.ped -g ch1.gen -m t1.trt -l ch1.loc

or, rely on a set file order like the following:

>mlod data.par data.ped t1.trt ch1.loc ch1.gen

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file, t1.trt is the name of the trait locus description file or type probability file, ch1.loc is the name of the marker locus description file, and ch1.gen is the name of the genome description file.

13.3.2 The mlod Block

A mlod block in the parameter file sets the options on how to perform an analysis using MLOD.

The following table shows the syntax for a mlod parameter which starts the mlod block.

Notes

1. An analysis output file is generated for each analysis performed. The name of this file may be provided in the out attribute of the mlod parameter . If no filename is provided, the filename defaults to the name of the region with the extension ".lod" appended to it.

The following table lists the parameters and attributes that may occur in a mlod block.

Notes

- 1. This causes the region to be analyzed using the current parameter settings and the corresponding output to be generated. If the value of the region parameter is not the name of a valid region, then the analysis is skipped. If multiple region parameters are specified in the same analysis block, then the last region specified will be used.
- 2. The scan_type parameter defines the locations where LOD scores are to be computed. If the value of scan_type is set to **marker**, then LOD scores are computed only at observed marker loci. If set to **interval**, then LOD scores are computed only at intervals between markers defined by the distance attribute. If set to both, then LOD scores are computed both at the marker locations and at the intervals defined by the distance attribute.
- 3. If output_pedigrees is set equal to marker, the output for each pedigree is printed only for the markers. If output_pedigrees is set equal to interval, it is printed only for the intervals defined by the distance attribute for the scan_type parameter. If set equal to both, all points in the region are produced. Note that the scan_type parameter needs to be set properly for this option to work. For example, when scan_type is set to **marker**, then output_pedigrees can be set to only either none or marker. When scan_type is set to both, then output_pedigrees can be set to any value.
- 4. If sample_detail is set equal to removed, the table only includes those individuals removed from analysis (with reasons for removal), if set equal to **all**, then all individuals are included in the table with reason for removal or being kept.

13.4 Program Output

MLOD produces several output files that contain results and diagnostic information:

13.4.1 Genome Information Output File

This file includes a table for each marker listing allele and genotype population frequencies, assuming Hardy-Weinberg equilibrium. If allele frequencies do not sum to 1.0, they are standardized to 1.0, so these frequencies may not be as described in the locus description files.

13.4.2 LOD Analysis Summary Output File

The LOD summary output file contains a table for each analysis performed by MLOD. These tables summarize LOD scores and information content for each point considered in the analysis by summing LOD scores from all pedigrees in the data set into a single LOD statistic. Information content is similarly summarized.

Example:

```
# Summary LOD score Output file
```

```
Analysis: Analysis 1 (chr5
```


13.4.3 LOD Analysis Detailed Output File

A separate LOD analysis output file is created for each analysis performed by MLOD. This file contains a table for each pedigree analyzed, listing LOD scores and information content at each marker for each trait analyzed. Points between markers are also be listed if the pedigree_lod_out parameter has been set to true.

Example:

. .

Chapter 14

PEDINFO

PEDINFO provides many useful descriptive statistics on pedigree structure including means, variances and tables of family, sibship and pedigree sizes, and counts of various types of relative pairs. Statistics based on trait phenotypic status (i.e., limited to traits not having missing values) are can also be requested.

14.1 Limitations

PEDINFO cannot correctly process a pedigree that contains loops; however, the program does indicate the presence of loops within the given pedigree data file.

14.2 Theory

14.2.1 Terminology

PEDINFO operates by iterating over the pedigree structures and keeps counts and distribution information of various elements. The following table defines some terms used in PEDINFO that are not defined elsewhere in this document:

14.2.2 Problematic Family Structures

A *marriage ring* is a chain of at least four spouses who form a cycle, for example, the founders in the following pedigree (individuals #1, 2, 3 and 4):

Individuals with multiple mates are enumerated in the output. For example, if the pedigree depicted above has Pedigree ID 1, the PEDINFO ouput will be:

These rings cause computational difficulties for current programs using full pedigree structure information, and therefore individuals with multiple mates are listed so that users can find and break these rings as they see fit.

Loops indicate either consanguineous (marriage between relatives) or other marriage loops, eg., two brothers married to two sisters:

Consanguineous and other marriage loops can also cause computational difficulties for current programs using full pedigree structure information and may also need to be broken. To facilitate this process, consanguineous matings are listed by pedigree and by the pair of relatives who have mated. For example, if the above pedigree has Pedigree ID 1, the PEDINFO output will be:

When there are marriage rings or loops in the pedigree, some pairs are not distinct and therefore the pair counts output by PEDINFO may not be accurate.

In the case of a consanguineous pedigree, the number of generations may be indeterminable and " undet" will appear in the generation statistics output by PEDINFO; e.g.

Breaking loops can be done either by duplicating individuals or by removing certain connecting individuals.

14.3 Program Input

14.3.1 Running pedinfo

A typical run of the PEDINFO program may use flags to identify the file types like the following:

>pedinfo -p parameters -d pedigree_data

or, rely on a set file order like the following:

>pedinfo parameters pedigree_data

where parameters is the name of the parameter file and pedigree_data is the name of the pedigree data file.

14.3.2 The pedinfo Block

A pedinfo block in the parameter file sets the options on how to perform an analysis using PED-INFO.

The following table shows the syntax for a pedinfo parameter which starts the pedinfo block.

The following table lists the parameters and attributes that may occur in a pedinfo block.

Notes

- 1. The each_pedigree parameter is used to specify whether results should be calculated for each pedigree separately in addition to a set of results for all the pedigrees taken as a whole.
- 2. By default PEDINFO generates a report of general pedigree structure information *without* regard to any trait. If the suppress_general parameter is set to true, then this general output is suppressed, and reports are given only with respect to some specified trait.
- 3. The trait and covariate parameters are used to specify trait or covariate variables for which statistics are to be calculated. The value of a variable parameter should be set to the name of a variable field read from the pedigree data file or created using a function statement. This parameter can occur more than once. To be included in the statistics an individual must not have a missing value for variables included here as traits or covariates. See Note 4 for details about how missing data are treated.

If a single binary variable is specified for analysis, counts of pairs that are concordant unaffected, discordant, concordant affected and uninformative will be displayed. If no trait or covariate variables are specified, only non-trait or covariate information (i.e., based on pedigree structure alone) will be used to determine counts.

Variable Type

4. The following table details the way missing data for the variable (or variables) in question are treated for various statistics as a function of variable type. For the multiple variable case, an individual must have non-missing values for each of the specified variables to be considered informative (non-missing)

Description

- (a) All sibships are counted regardless of whether the sibs have missing data.
- (b) All sibships are counted, but sibship size refers to number of sibs with data.
- (c) All pedigrees are counted, but pedigree size refers to number of pedigree members with data.
- (d) All pedigrees are counted, but for a nuclear family to be counted it must have at least one parent and one child without missing data.
- (e) Each pair is included in exactly one category.
- (f) Only pairs where both individuals have non-missing data are included.
- (g) Each individual is included in exactly one category.
- (h) Only individuals without missing data are included.

The following are all valid pedinfo statements and could all occur within the same parameter file:

```
# A pedinfo statement that runs with all default values
pedinfo
pedinfo
\mathbf{f}}
# A pedinfo statement that specifies the name of an output file
# and requests a separate report for each pedigree
pedinfo,out=allpeds
\mathfrak{t}each_pedigree=true
}
# A pedinfo statement that specifies 2 traits, for each of which an individual
# must have no missing data to be included in the trait-specific pedigree statistics
pedinfo,out=analysis1
{
   trait=A \qquad # if these two traits are binary
```

```
trait=hematocrit
}
# A final example
 pedinfo,out=output
 {
  covariate=B
  each_pedigree=true
}
```
14.4 Program Output

Output files produced by PEDINFO containing results and diagnostic information are:

14.4.1 Analysis Output File

The PEDINFO analysis output file may contain the following types of tables (See notes following pedinfo parameter block for more information.):

- Tables of statistics pertaining to the structure of all of the data as a whole.
- Tables of statistics pertaining to the structure of a single pedigree.
- Tables of statistics pertaining to a specific variable or set of variables for the data as a whole.
- Tables of statistics pertaining to a specific variable or set of variables for a single pedigree.

Example:

Chapter 15

RELPAL

This is a regression-based univariate or multivariate model-free two-level Haseman-Elston linkage program that models trait data from relative pairs as a function of marker allele sharing identityby-descent (IBD) as proposed by Wang and Elston (2005, 2006). Available analyses can use both single- and multi- point IBD information, and models allow for both binary and quantitative traits caused by segregation at multiple genetic loci, including one epistatic interaction and covariate effects.

15.1 Limitations

This program is limited to pedigrees without loops and does not generate IBD sharing estimates itself. That must be done using GENIBD, which outputs an IBD sharing file as input for RELPAL. It is assumed that the only bilineal relatives in the data are full sibs.

15.2 Theory

15.2.1 Basic Notation

Let the number of pedigrees in the analysis be *K* .

Let *i* be the index of an individual: $i = 1, 2, ..., m_k$, where m_k is the total number of individuals in the *k* -th family.

Let the number of informative relative pairs in the k -th family be n_k , $k = 1, 2, ..., K$.

Let *j* be the index of a relative pair: $j = 1, 2, ..., \sum_{k} n_k = n$, where *n* is the total number of relative pairs.

Let the number of traits in the analysis be L, and let l be the index of a trait: $l = 1, 2, ..., L$.

Let ⊗ denote the Kronecker product.

Conditional on the marker information available, at a particular genomic location let \hat{f}_{1j} be the probability of sharing 1 allele IBD, and \hat{f}_{2j} be the probability of sharing 2 alleles IBD, for the *j*-th relative pair. Note that $\hat{f}_{2i} = 0$ in the case of non-full sib pairs.

Let $\pi = (1 + w_1)/4$ and $\hat{\pi}_j = \hat{f}_{2j} + w_1 \hat{f}_{1j}$ where $0 \le w_1 \le 0.5$ (Whittemore and Tu, 1998), for the *j* -th relative pair. The current default value of w_1 is 0.5.

15.2.2 Univariate Two-level Haseman-Elston Regression Model

With the assumption of randomly sampled pedigrees, a general two-level trait model is given by

$$
y_{ik} = x_{ik}\beta + z_{ik}b + e_{ik}
$$

where

- y_{ik} is the trait value of individual *i* in pedigree *k*,
- $x_{ik} = (1, x_{ik2}, ..., x_{ikr})$ is the design vector for fixed effects at the individual, or first level,
- $\beta = (\beta_1, \beta_2, ..., \beta_r)^T$ is the coefficient vector of fixed effects at the first level,
- $z_{ik} = (z_{iku_1}, ..., z_{iku_s}, z_{ikp})$ is the design vector for random effects at the pedigree, or second level,
- *b* is the coefficient vector at the second level containing coefficients $u_1...$ u_s for up to *s* covariates at the pedigree level and a polygeneic effect *pik*,
- e_{ik} are random individual effects assumed to be independently and identically distributed as N(0, σ_e^2).

The effect of a QTL is incorporated into either the first level or the second level, depending on the particular analysis. The polygenic effects are assumed to be independent across all founders in all pedigrees, and any common environmental effects are assumed to be confounded with the polygenic effects.

Under this trait model, we have $r + 1$ coefficients of fixed effects β at the first level, *s* variances $\sigma_u^2 = (\sigma_{u_1}^2, ..., \sigma_{u_s}^2)$ and the variance σ_p^2 at the second level, and σ_e^2 . The QTL parameter σ_g^2 may be included in β or σ_u^2 . However, in a linkage analysis, the number of disease alleles is not observed directly (or the marker cannot be assumed to be in linkage disequilibrium with the disease locus), so we model the QTL effect at the second level, *i.e.*, $\sigma_{u_1}^2 = \sigma_g^2$.

15.2.2.1 Estimation

To estimate the above parameters, $(\sigma_u^2, \sigma_p^2, \sigma_e^2)$, an iterative generalized least squares (IGLS) algorithm (Goldstein, 2003) is used as follows:

- 1. First, an ordinary least squares estimate $\hat{\beta}$ of the fixed effect parameter β is obtained with the assumption that (σ_u^2, σ_p^2) are 0.
- 2. Let

$$
\tilde{e}_k = y_k - X_k \hat{\beta}
$$

where the length of \tilde{e}_k , y_k , and X_k is the number of individuals in pedigree k . Then, the expectation of the cross-product matrix $\tilde{e}_k \tilde{e}_k^T$ is simply the variance-covariance matrix for pedigree k, i.e., $E(\tilde{e}_k \tilde{e}_k^T) = V_k = Z_k \Omega Z_k^T + \sigma_e^2 I$, where the way the entries in Z_k for pedigree *k* are defined depends on the linkage analysis, Ω is a diagonal matrix with diagonal elements (σ_u^2, σ_p^2) , and *I* is an identity matrix. Residuals, \tilde{e} , are transformed to follow a marginal normal distribution with the same varianes, and this normalization can be optionally turned off. We now rearrange the cross-product matrix $\tilde{e}_k \tilde{e}_k^T$ and the variance-covariance matrix V_k as vectors by stacking the columns one on top of the other so that we have $E[vec(\tilde{e}_k \tilde{e}_k^T)] =$ $vec(V_k)$. This can be written as

$$
\tilde{\pmb{\eta}}_k = \Delta_k \pmb{\varpi} + \pmb{\varepsilon}_k
$$

where
$$
\tilde{\eta}_k = vec(\tilde{e}_k \tilde{e}_k^T), \ \boldsymbol{\varpi} = (\sigma_e^2, \sigma_u^2, \sigma_p^2), \ \Delta_k = \begin{bmatrix} 1 & z_{1ku} z_{1ku} & z_{1kp} z_{1kp} \\ 0 & z_{1ku} z_{2ku} & z_{1kp} z_{2kp} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_{mku} z_{mku} & z_{mkp} z_{mkp} \end{bmatrix}
$$
 is the de-

sign matrix of cross-products for pedigree k , and ε_k is a residual vector. Then, the left-hand side of the equation can be treated as the response vector in a linear model, the right-hand side contains observed explanatory variables in the design matirx Δ_k , and the entries of $\bar{\omega}$ are the regression coefficients of this linear model. A generalized least squares analysis is used to estimate the entries of the expectation of $\bar{\omega}$, namely

$$
\tilde{\boldsymbol{\varpi}} = (\Delta^T V^{*-1} \Delta)^{-1} \Delta^T V^{*-1} \tilde{\boldsymbol{\eta}}
$$

where is V^* a block-diagonal matrix made up of the matrices V^* _k = 2($V_k \otimes V_k$) and $\tilde{\sigma}$ = $(\hat{\sigma}_e^2, \hat{\sigma}_u^2, \hat{\sigma}_p^2)^T$.

3. Based on $\tilde{\varpi}$, the variance-covariance matrix \hat{V} , an estimate of *V*, the block diagonal matrix made up of the matrices V_k , is obtained. Then, the weighted least squares estimate of the vector of fixed coefficients is given by

$$
\tilde{\beta} = (X^T \hat{V}^{-1} X)^{-1} X^T \hat{V}^{-1} y.
$$

Step 1 to 3 are iterated until the procedure converges.

4. Finally, with the assumption of multivariate normality, the model-based estimate of the variancecovariance matrix of $\tilde{\boldsymbol{\omega}}$ is given by $Var(\tilde{\boldsymbol{\omega}}) = (\Delta^T \hat{V}^{*-1} \Delta)^{-1}$.

15.2.3 Multivariate Two-level Haseman-Elston Regression Model

The multivariate model for general unlooped pedigrees is constructed under the same framework of two-level regression as the univariate case above. Denote the vector of trait values observed on family *k*, of length lm_k , y_k with transpose y_k^T . After the trait values have been adjusted for individual covariates, $E(y_k y_k^T)$ is the variance-covariance matrix of the traits for family *k*.

The variance-covariance matrix is given by

$$
P\otimes\Phi_k+G\otimes\Pi_k+E\otimes I_k
$$

where

- Φ_k is an $m_k \times m_k$ matrix of coefficients of relationship between relative pairs,
- Π_k is an $m_k \times m_k$ matrix of the observed proportions of alleles shared IBD at a particular locus,
- I_k is an identity matrix,
- *P* , *G* and *E* are *L* × *L* matrices of polygenic, quantitative trait locus (QTL) and random individual effects, respectively.

Let $E(y_k y_k^T) = E(A)$, let the variance-covariance matrix under the null hypothesis of no QTL effect be B^0 , and let the variance-covariance matrix under the alternative hypothesis be B^1 . Then, we have $vec[E(A)] = vec[B¹]$ at the second level regression, *i.e.* rearranging the cross-product matrix $E(A)$ and the variance-covariance matrix $B¹$ as vectors by stacking the columns on top of each other. The parametrers of this regression model are obtained by applying an IGLS algorithm as in the univariate case.

Under this regression model, a score is defined by

$$
U = \sum_{k=1}^K U_k = \sum_{k=1}^K D_k^T W_k^{-1} S_k
$$

where

- $S_k = vec(A_k)$, which measures the trait similarity among relative pairs,
- $D_k = \frac{\partial vec(B_k^1 B_k^0)}{\partial \sigma_k}$ $\frac{\partial f_k - \partial f_l}{\partial \sigma_g}$, which measures the genetic (i.e. the proportion of alleles IBD) similarity among relative pairs and in which σ_g is a vector of QTL parameters of interest,
- W_k is an appropriate weight matrix to take account of the correlations between elements in S_k , which is defined as $W_k = 2(\hat{B}_k^0 \otimes \hat{B}_k^0)$ in our analysis.

U is further modified as a weighted least squares estimate of *A* that uses a weight matrix estimated under the null hypothesis, which is $U^* = (\sum_{k=1}^K D_k^T W_k^{-1} D_k)^{-1} U$, and we finally define a score statistic by

$$
T_{unad\,justed} = U^{*T} \Sigma_{U^*}^{-1} U^*
$$

for some Σ_{U^*} defined below. In the case of a univariate trait, this statistic asymptotically follows a 50:50 mixture of a $\chi^2_{df=1}$ and a point mass at 1. However, for multivariate models, the asymptotic distribution of this statistic may be very complex and the p-value difficult to obtain. For this reason, we evaluate the p-value through a Monte Carlo procedure described in the next section.

15.2.3.1 One-sided Adjusted Score Statistic

Score tests for testing variance-covariance components under constrained parameterization requires replacing the classical score test statistic by an appropriate one-sided version as in Verbeke and Molenberghs (2003). Here we use the Cholesky decomposition to find feasible estimates, and the score statistic T is adjusted as

$$
T_{adjusted} = U^{*T} \Sigma_{U^*}^{-1} U^* - \inf_{b \in \Theta} ((U^* - b)^T \Sigma_{U^*}^{-1} (U^* - b)),
$$

where Θ is positive definite.

15.2.3.2 Variances

Four different estimators of the variance of U^* are given.

For the naive approach, assuming that the data generating process is multivariate normal, the estimator is given by

$$
\Sigma_{naive} = (\sum_{k=1}^{K} D_k^T W_k^{-1} D_k)^{-1}.
$$

The standard sandwich estimator under the null hypothesis is given by

$$
\Sigma_{null} = \Sigma_{naive} (\sum_{k=1}^{K} U_k U_k^T) \Sigma_{naive}.
$$

An alternative estimator is given by

$$
\Sigma_{alt} = \Sigma_{naive} \left(\sum_{k=1}^{K} (U_k - \hat{E}(U_k)) (U_k - \hat{E}(U_k))^{T} \right) \Sigma_{naive}
$$

where $\hat{E}(U_k) = D_k^T W_k^{-1} D_k U^*$.

A new estimator using the variance of the IBD values conditional on Y is given by

$$
\Sigma_{ibd} = \Sigma_{naive} (\sum_{k=1}^{K} B_k Var[Vec(\tilde{\Pi}_k)]B_k^T)\Sigma_{naive}
$$

where there exists a matrix B_k such that $U_k = B_k Vec(\tilde{\Pi}_k)$.

15.2.4 Significance Tests

15.2.4.1 First level Wald Test

For both the univariate and multivariate cases, the significance of the effects at the first level are tested as follow. The weighted least squares estimate of the vector of fixed coefficients at the first level is given by

$$
\tilde{\beta} = (X^T \hat{V}^{-1} X)^{-1} X^T \hat{V}^{-1} y,
$$

and

$$
\tilde{\beta} \sim MVN[\beta, (X^T\hat{V}^{-1}X)^{-1}].
$$

We wish to test $H_0: A\underline{\beta} = 0$ vs. $H_1: A\underline{\beta} \neq 0$. Then $\chi^2_{d f = F} = (A\underline{\beta})^T [A(X^T\hat{V}^{-1}X)^{-1}A^T](A\underline{\beta})$, where *A* is a matrix whose elements are 0 and 1 to select only the effects *F* being tested.

15.2.4.2 Second level Score Test

For both the univariate and multivariate cases, the asymptotic p-value for the adjusted score statistic *T* is calculated as follows.

Let *Torg* be the adjusted score statistic from the data.

Let F_c be the cumulative distribution function of a $\chi^2_{d f=c}$ where *c* is the number of variancecovariance components in the test.

- 1. Simulate a vector of size *c* from a multivariate normal distribution, then divide the vector by its norm to get *U*.
- 2. Calculate *T*, the adjusted score statistic, using *U* as a score.
- 3. Repeat steps 1 and 2 *N* times, resulting in T_1 , T_2 , ..., T_N . Then the estimated p-value is $\hat{p}_N = \frac{\sum_i (1-F_p(\frac{T_{org}}{T_i}))}{N}$ $\frac{P(Y_i|Y_i)}{N}$.
- 4. If not converged, then return to step 1. Otherwise, stop.

N is determined such that the estimated asymptotic p-value \hat{p} is within a proportion ω (the width parameter) of its true p-value *p* with predetermined confidence probability γ (the confidence parameter). That is, we want the standard deviation $s_{\hat{p}}$ of \hat{p} to be proportional to p , where the sample variance of \hat{p} is $s_{\hat{p}}^2 = \frac{s^2}{N}$ $\frac{s^2}{N}$, where *s*² is the variance of the *N* values of $1 - F_p(\frac{T_{org}}{T_i})$ $\frac{org}{T_i}$). So we choose *N* such that $Pr(|\hat{p} - p| \le \omega \hat{p}) = \gamma$. Using a normal approximation for the distribution of \hat{p} , we obtain

$$
N = \left(\frac{s^2}{\hat{p}\omega^2} [\Phi^{-1}(\frac{\gamma+1}{2})]^2\right)
$$

where Φ is the standard normal cumulative distribution function.

15.3 Program Input

15.3.1 Running relpal

A typical run of the RELPAL program may use flags to identify the file types like the following:

>relpal -p data.par -d data.ped -i ch1.ibd

or, rely on a set file order like the following:

>relpal data.par data.ped ch1.ibd

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file, and ch1.ibd is the name of the IBD sharing file.

15.3.2 The relpal Block

A relpal block in the parameter file sets the options on how to perform an analysis using SIBPAL. The following table shows the syntax for a relpal parameter which starts the relpal block.

The following table lists the parameters and attributes that may occur in a relpal block.

Notes

- 1. The value of a trait parameter should be set to the name of a trait or covariate field read from the data file or created by means of a function block. This parameter may be included more than once. To have a valid analysis block, at least one trait parameter must be specified. Depending on the number of trait parameters in the analysis block, either a univariate or multivariate analysis will be performed.
- 2. The value for a model parameter should be set to one of the following values:
	- zero_marker : Haseman-Elston regression model with covariate(s) only. Note that no ibd data are used with this option. In this model, at least one covariate parameter has to be specified as a test covariate in the second_level sub-block to perform a score test at the second level. If more than one covariate parameter is specified in the second_level sub-block, one covariate parameter has to be specified as a test covariate. Then the test of this covariate effect in the presence of other(s) will be performed. When no test covariates are listed in the second_level sub-block, only the tests in the first level are performed, without any score test in the second level.
	- single_marker : Haseman-Elston regression model with one marker and covariate(s). In this model, all markers/locations in the IBD sharing file are test markers by default. So, no valid marker parameters are required for this model since the analyses are done using all markers/locations in the IBD sharing file one by one when no valid marker parameters are listed in the second_level sub-block. If a subset of the markers from the IBD file is to be analyzed, one or more marker parameters should be specified. Any number of optional non-test covariate parameters may be included in the second_level sub-block for this model.
	- multiple_marker : Haseman-Elston regression analysis with multiple markers and covariate(s). At least one marker parameter has to be specified in the second_level sub-block for this model. If one marker parameter is specified as a test marker, then the tests for linkage to this marker are done in the presence of other marker using all markers in the IBD sharing file one by one. If one marker parameter is specified as a non-test marker, then the tests for linkage to all markers in the IBD sharing file are done one by one in the presence of this marker. If more than one marker parameters is specified, one marker parameter has to be specified as a test marker. Then the test for linkage to this test marker in the presence of other(s) will be performed. Any number of optional non-test covariate parameters may be included in the second_level sub-block for this model.

15.3.2.1 The first_level Sub-Block

The following table lists the parameters and attributes that may occur in a first_level sub-block.

Notes

1. The value of a covariate parameter should be set to the name of a trait or covariate field read from the data file or created by means of a function block. The covariate parameter may be included more than once. If no valid covariate parameters are listed, then by default no covariates are included in the first level. If the adj_trait attribute is specified with a valid name of a trait, then only the specified trait among all traits in the model is adjusted for this covariate. Otherwise all traits in the current analysis are adjusted for the covariate in the first level regression. Note that this adj_trait attribute is only applicable in a multivariate analysis.

2. The interaction parameter should contain a sub-block of two first level covariate parameters to specify a multiplicative interaction term in the model.

```
interaction
{
   covariate = AGEcovariate = BMI
}
```
- 3. If the normalize_residual parameter is set to false, then the residuals from the first_level will not be normalized before calculation of variance components at the second_level. If no valid normalize_residual parameter is listed, or if the parameter is set to true, then by default the residuals from the first_level will be transformed to follow a marginal normal distribution with the same variance before being used at the second_level.
- 4. If the batch parameter is specified in the sub-block, then all first level covariate fields read from the data file or created by means of a function block will be automatically included one by one. Note that this option is valid only with the zero_marker model and when no second level test covariates are specified.

15.3.2.2 The second_level Sub-Block

The following table lists the parameters and attributes that may occur in a second_level subblock.

Notes

1. The value of a marker parameter should be set to the name of a marker for which IBD sharing information was generated and stored in the IBD sharing file. If the test attribute is specified in the case of multiple_marker model, then this marker is used to test for linkage in the presence of other marker(s). If no valid marker parameters are listed then all markers in the IBD sharing file are used one by one.

- 2. The value of a covariate parameter should be set to the name of a trait or covariate field read from the data file or created by means of a function block. The covariate parameter may be included more than once. If the test attribute is specified in case of zero_marker model, then this covariate is used to test the effect in the presence of other covariate(s) . If no valid covariate parameters are listed, then by default no covariates are included.
- 3. The interaction parameter should contain a sub-block of two main effect parameters; two marker parameters, or one second level covariate parameter and one marker parameter that specify a multiplicative interaction term in the model. Note that this option is only valid in a univariate analysis with both corresponding main effects included in the analysis, so marker by marker interaction can be included only in multiple_marker model. The following interaction sub-block specifies a gene-environment interaction term between D1S344 and BMI:

```
interaction
{
  market = D1S344covariate = BMI
}
```
4. If the batch parameter is specified with only one covariate or marker parameter in the sub-block, then the interaction term between the given covariate or marker and a marker in the IBD sharing file will be automatically included one by one. Note that this option is valid only in a univariate analysis. The following interaction sub-block specifies a geneenvironment interaction term between the dominance component of D1S344 and the squared BMI difference:

```
# Include marker by covariate interaction for all markers.
interaction
{
   covariate = BMI
}
# Include marker by marker interaction for all markers.
interaction
{
   marker = D1S344}
```
15.3.2.3 The data_options Sub-Block

The following table lists the parameters and attributes that may occur in a data_options subblock.

Notes

- 1. The trait specified by a subset parameter should be a binary trait coded as 0 for individuals to be excluded from, and 1 for individuals to be included in, the analysis. The subset parameter may be included more than once, in which case the only individuals included in the analysis are those for which all the indicated binary traits are coded 1.
- 2. The value of a use_pairs parameter should be set to one of fsib, sib or all depending on whether the analysis include full sib pairs only, full and half sib pairs, or all relative pairs from the IBD file.

15.3.2.4 The output_options Sub-Block

The following table lists the parameters and attributes that may occur in a output_options subblock.

15.3.2.5 The pvalue_options Sub-Block

The following table lists the parameters and attributes that may occur in a pvalue_options subblock.

Notes

1. If not specified, different seeds, and hence different results, will be obtained each time a given analysis is performed.

15.4 Program Output

RELPAL produces several output files that contain results and diagnostic information:

15.4.1 Summary Output File

One main analysis output file, named "relpal.out", is generated per run of RELPAL. It contains the results of all tests.

Example:

```
====================================================================================
 Two-Level Haseman-Elston Regression Analysis for General Pedigree
 - Second (Pedigree) Level Score Test Summary Output
====================================================================================
Traits : (1) - EF, Quantitative
          (2) - Q2, Quantitative
Legend :
   * - significance .05 level
    ** - significance .01 level
   *** - significance .001 level
   nai - naive variance
   sdw - robust sandwich variance
   alt - alternative variance
   ibd - allele sharing variance
   # - The number of pedigrees is too small for this result to be
          reliable when analyzing this number of traits.
Empirical p-value options used :
   seed = 0min replicates = 20
   max replicates = 10000
```


15.4.2 Detailed Output File

One detailed output file, named "relpal.det", is generated per run of RELPAL optionally. It contains more detailed information of the results of all tests.

Example:

```
====================================================================================
 Two-Level Haseman-Elston Regression Analysis for General Pedigree
  - Second (Pedigree) Level Score Test Summary Output
====================================================================================
Traits : (1) - EF, Quantitative
          (2) - Q2, Quantitative
Legend :
   * - significance .05 level
    ** - significance .01 level
   *** - significance .001 level
   nai - naive variance
   sdw - robust sandwich variance
   alt - alternative variance
   ibd - allele sharing variance
   # - The number of pedigrees is too small for this result to be
         reliable when analyzing this number of traits.
Empirical p-value options used :
   seed = 0min replicates = 20
```
max replicates = 10000 == Test 1 # == ----- Model ----- H0: EF, Q2 ~ [Intercept + Q3] + POLYGENIC_EFF + RANDOM_EFF H1: EF, Q2 ~ [Intercept + Q3] + D5G1 + POLYGENIC_EFF + RANDOM_EFF ------ Sample ------ Number of individuals used at first level $= 686$ Number of relative pairs used at second level = 771 ----------------- Estimates from H0 ----------------- Test Nominal Variable Estimate Variance-Covariance Chi-sq. P-value ------------------------- ------------ ------------------- --------- --------- Q3(1) 0.0772 0.0001 0.0000 242.5299 0.0000000 *** Q3(2) 0.2275 0.0000 0.0002 ------------------ Score Test Results ------------------ Test Unadjusted Adjusted Empirical Number of Variable Var T-value T-value P-value Replicates ------------------------- --- ---------- ---------- --------- ---------- D5G1 nai 0.1740 0.0085 0.8281762 22 sdw 0.4172 0.0184 0.9467879 26 alt 0.1740 0.0085 0.8281762 43 ibd 0.2268 0.0301 0.9851194 24 == Test 2 == . . .

Chapter 16

RELTEST

RELTEST helps classify pairs in a sib pair linkage study according to their true relationship using autosomal genome scan data. It is based on a Markov process model of allele-sharing along chromosomes. The program currently performs analyses to classify putative sib pairs, putative half-sib pairs, putative parent-offspring pairs, and putative marital pairs into five different types of pairs: MZ twin pairs, full sib pairs, half sib pairs, parent-offspring pairs, and unrelated pairs. A summary file is produced that contains the identifiers of the putative full-sib pairs to be reclassified and their sibling and parent-offspring classification statistics; for each pair, missing data rates over the markers used; and histograms of the sibling classification statistic and parent-offspring classification statistic. An optional output file contains the same pair-specific statistics, but for all putative pairs other than MZ twins (i.e., putative half-sib, parent-offspring and unrelated pairs).

16.1 Limitations

The probability of misclassification depends on the total length of the genotyped genome provided and overall marker informativeness. The misclassification rates are minimal when at least half the genome is genotyped using microsatellite markers at most 20 cM apart. Individual pairs may be misclassified if one or both members have a high proportion of missing genotypes, as the classification cut points are based on the length of the genotyped genome and marker informativeness calculated for the entire sample. It should also be noted that the proportion of missing genotypes is calculated using as the denominator the number of markers listed in the genome file.

16.2 Theory

This program is intended primarily for late-onset diseases, for which parents are not typed and the number of typed sibs is often two. In this case, one cannot detect errors in relationship by looking for inconsistencies, and one must use the entire genome (or as much of it as possible) to examine the overall allele-sharing between the sibs. In practice, this program can be used for other types of data sets, and even pairs with late-onset disease will sometimes have typed parents or additional sibs. However, we do not use all the marker information to construct the relationship statistics. For each pair, only the marker information for that pair is used, and none from the other relatives, such as other sibs and parents. Pair-wise allele-sharing is computed using a multipoint algorithm.

16.2.1 Full Sib Pairs

Let \hat{f}_{jis} be the estimated probability that sib pair *j* shares *i* marker alleles identical-by-descent (IBD) at location *s* on a chromosome. We assume throughout that these IBD probabilities are obtained using multi-point methods. Feingold et al. (1993) proposed a Gaussian process model to describe the ideal (i.e., infinitely dense, fully informative) process for the estimated mean number of alleles shared IBD in a sample of N sib pairs at location *s*:

$$
X_s = \sum_{j=1}^N (\hat{f}_{j1s} + 2\hat{f}_{j2s}).
$$

If the marker is fully informative, X_s is the total number of alleles shared IBD in the sample at location *s*.

For the ideal process and a large sample of randomly sampled sib pairs, the mean-sharing statistic

$$
Z_s = (X_s - N) / (N/2)^{1/2}
$$

has mean equal to 0, variance equal to 1, and approximate Gaussian process covariance function exp(- β |*t*|), where *t* is the distance between markers and β =0.04 for sib pairs (Feingold et al., 1993). The parameter $β$ is a function of the recombination process and assumes that crossovers are independent, i.e., that there is no crossover interference.

Here we consider a single random sib pair *j*, and let Z_{js} be the mean-sharing statistic for a single pair (*N*=1). We obtain a measure of the average number of alleles shared by this pair over the entire genome. Let $k=1,2,\ldots,22$ index the human autosomes and L_k be the length of chromosome k in cM. The statistic

$$
Y_{jk} = \frac{1}{L_k} \int_0^{L_k} Z_{js} ds
$$

has expectation

$$
E(Y_{jk}) = \frac{1}{L_k} \int_0^{L_k} E(Z_{js}) ds = 0
$$

and variance

$$
Var(Y_{jk}) = \frac{1}{L_k^2} \int_0^{L_k} \int_0^{L_k} Cov(Z_{js}, Z_{jr}) dr ds = \frac{2}{\beta L_k} - \frac{2}{(\beta L_k)^2} (1 - e^{-\beta L_k})
$$
(16.1)

(Parzen, 1962; Olson, 1999). In the ideal case of fully informative, infinitely dense markers, the statistic Y_{ik} is the difference between the proportions of the chromosomes sharing 2 and 0 alleles IBD. More generally, it is the difference between the absolute areas above and below the null mean (sharing 1 allele IBD), divided by the length of the chromosome.

If putative sib pair *j* is a true sib pair, then *Yjk*/[Var(*Yjk*)] ¹/² has a standard normal distribution as $L_k \rightarrow \infty$. In practice, the normal approximation is somewhat inadequate for single chromosomes of modest length. A genome-wide measure, the sibling classification statistic given by

Figure 16.1: Approximate Mean-Corrected Allele Sharing

$$
Y_j = \left(\sum_{k=1}^{22} Y_{jk}\right) / \left[\sum_{k=1}^{22} Var(Y_{jk})\right]^{1/2},
$$

is well approximated by a standard normal distribution in the fully informative, infinitely dense case. Similarly, for any number of chromosomes *K*,

$$
Y_j = \left(\sum_{k=1}^K Y_{jk}\right) / \left[\sum_{k=1}^K Var(Y_{jk})\right]^{1/2}.
$$

Relationship estimation for each pair j in the sample is based on estimating genome-wide Y_j for each of the sib pairs. These statistics can be obtained in practice using a standard algorithm to calculate multipoint IBD at equally spaced points throughout the genome. For each chromosome, the absolute areas above and below the estimated mean-corrected allele-sharing curve is approximated using rectangles (see Figure [16.1\)](#page-262-0),

which is equivalent to computing:

$$
\hat{Y}_{jk} = [c\sqrt{2}\sum_{s=1}^{P} (X_{sj} - 1)]/P,
$$

where P is the number of points at which allele-sharing is computed and c is the distance (cM) between points (i.e., the width of the rectangles in Figure [16.1\)](#page-262-0).

16.2.2 Parent/Offspring Pairs

Parent/offspring pairs are always expected to share exactly one allele IBD, and so \hat{Y}_j cannot be used to discriminate between sib pairs and parent/offspring pairs. Therefore, a second Markov process

$$
X_{s}^{*}=-(\hat{f}_{j2s}+\hat{f}_{j0s}-\hat{f}_{j1s}).
$$

For a fully informative location *s*, the Gaussian process statistic

$$
Z_s^* = \sum_{j=1}^N \frac{X_s^*}{N^{1/2}}
$$

has a standard normal distribution in a large sample of sib pairs, with covariance function exp(- β |*t*|), where now β = 0.08. The new statistic Y_j^* , the parent offspring classification statistic, is calculated in the same manner as before, i.e.,

$$
Y_j^* = \left(\sum_{k=1}^K Y_{jk}^*\right) / \left[\sum_{k=1}^K Var(Y_{jk}^*)\right]^{1/2},
$$

with

$$
\hat{Y}_{jk}=[-c\sum_{s=1}^P X_{sj}^*]/P,
$$

and the variance is calculated using equation [16.1](#page-261-0) with β = 0.08.

16.2.3 Incomplete Marker Information

When markers are not infinitely dense and fully informative, the variances of the Sibling and Parent-Offspring Classification Statistics are less than one. Classification criteria (cut points) may be determined using the overall marker informativity and the length of the genotyped genome. The *Average Marker Information Content* (AMIC) (Kruglyak and Lander 1995)

is defined as

$$
AMIC = \sum_{p=1}^{M} r^2(s)/M,
$$

where *M* is the total number of points over which the genome IBD probabilities are calculated and

$$
r^{2}(s) = 1 - \frac{\sum_{i=1}^{N} \sigma_{i, residual}^{2}(s)}{\sum_{i=1}^{N} \sigma_{i, initial}^{2}} = 1 - \frac{2 \sum_{i=1}^{N} \sigma_{i, residual}^{2}(s)}{N},
$$

N is total number of sib pairs in the sample and $\sigma_{i, residual}^2(s)$ is the variance of the IBD distribution at point *s* for sib pair *i*.

16.2.4 Classification Cut Points

The best-fit regression equations for obtaining classification cut points, are:

- $\log_{10}(-C_u) = 0.421 + 0.506 \log_{10}(T) + 1.162 \log_{10}(AMIC) + 0.472(\log_{10}(AMIC))^2$,
- $\log_{10}(-C_h) = 0.141 + 0.524 \log_{10}(T) + 0.237 \log_{10}(AMIC) 0.861(\log_{10}(AMIC))^2$,
- $\log_{10}(-C_p) = 0.2 + 0.518 \log_{10}(T) + 2.220 \log_{10}(AMIC)$,
- $C_m = 3.27$,

where *T* is the total length of the genotyped genome in cM divided by 150, and C_u , C_h , C_m , and C_p are the classification cut points for unrelated pairs, half sib pairs, MZ twins, and parent offspring pairs, respectively. C_u , C_h , C_m are used to classify pairs on the basis of the sibling classification statistic into unrelated, half sibs, full sibs, and MZ twins. C_p is used to classify pairs into full sib and parent-offspring pairs on the basis of the parent-offspring classification statistic.

16.2.5 Strategy for Classifying Putative Full-Sib and Non-Full-Sib Pairs

There are two steps to classify each pair:

- 1. Using Y_i and the cut points defined above, we classify as follows: Unrelated $\leq C_u$ < Half sib < C_h < Sib < C_m < MZtwin
- 2. If the pair is classified as a sib pair in step 1, we use Y_j^* and the parent_offspring cut point: Parent/offspring $\leq C_p <$ *Sib*

16.2.6 Nonparametric Estimation Procedure

After calculating the Y_j and Y_j^* , a nonparametric estimation procedure is used to obtain the mean and variance of the sib-pair distributions of these two sets of statistics.

1. Estimating the shift for Y_j :

We use the L_2 -error procedure (Scott, 2000) to maximize the function

$$
\frac{2}{n}\sum_{j=1}^n \phi(Y_j|\mu,\sigma^2) - \frac{1}{2\sqrt{\pi\sigma^2}},
$$

where μ and σ^2 are parameters, *n* is the total number of sib pairs (all putative full sib pairs), and $\phi(.)$ is the normal density function

$$
\phi(Y_j|\mu,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(Y_j-\mu)^2}.
$$

2. We then adjust the cut points:

New Cut point = Old Cut point + μ from step 1.

- 3. We repeat the same steps 1 and 2 for the Y_j^* obtained from all putative full sib pairs.
- 4. We perform the classification as described in [16.2.5](#page-264-0) using the new cut points.

To test the deviation of the sib pair mean from zero, we use the Y_i from putative full sib pairs now classified as true sib pairs to compute the mean

$$
\bar{Y} = \frac{\sum_{j=1}^{n} Y_j}{n}
$$

and the standard error of the mean

$$
S.E.(\bar{Y}) = \frac{1}{\sqrt{n}} \sqrt{\left(\sum_{j=1}^{n} Y_j^2 - \frac{\left(\sum_{j=1}^{n} Y_j\right)^2}{n}\right)/n} .
$$

Then a confidence interval is constructed as

$$
\bar{Y} \pm 2S.E.(\bar{Y}).
$$

If zero is not included in this interval, a warning is printed in the output. The user should at this point note that the sib-pair histogram is shifted significantly (in the statistical sense) away from its null hypothesis mean value of zero. If such significant deviation is substantial, there may be largescale error in the data or specification of parameters. Our previous experience with real data sets has shown that such error may be due to

- 1. Gross misspecification of marker allele frequencies,
- 2. Misalignment of marker description information between the parameter file, the data file and/or the genome file, and/or
- 3. Large-scale genotype errors.

Examples of large-scale genotype errors that have caused large " shifts" in the sib-pair histogram have included:

- 1. Errors in programs translating data from the genotyping lab to the pedigree data file and
- 2. Extensive binning errors in the assignment of genotypes.

The above list includes only errors we have been alerted to by RELTEST; other sources of error detectable by RELTEST are clearly possible. We suggest using RELTEST not only to classify pairs according to relationships, but also as a general test of the overall accuracy of the data and parameter specifications (Olson et al., 2004).

16.3 Program Input

16.3.1 Running reltest

A typical run of the RELTEST program may use flags to identify the file types like the following:

>reltest -p data.par -d data.ped -l m.loc -g g.map

or, rely on a set file order like the following:

>reltest data.par data.ped m.loc g.map

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file, m . loc is the name of the marker locus description file, and g map is the name of the genome description file.

16.3.2 The multiple pedigree Block

RELTEST can read multiple pedigree data files in cases where each pedigree data file contains the markers for a single chromosome. For each pedigree data file, there has to be a corresponding pedigree block with file name specified. All other fields should be the same, except for the marker fields, for all pedigree data files used.

Example:

```
pedigree, file=ped1
{
.
.
.
marker="ch1m1"
.
.
.
}
```
pedigree, file=ped2 { . . . marker="ch2m1" . . . } pedigree, file=ped3 { . . . marker="ch3m1" . . . }

16.3.3 The reltest Block

A reltest block in the parameter file sets the options on how to perform an analysis using REL-TEST.

The following table shows the syntax for a reltest parameter which starts the reltest block.

The following table lists the parameters and attributes that may occur in a reltest block.

Notes

- 1. By default, all four pair types will be analyzed.
- 2. Normally, cut points are automatically generated based on the pedigree data, as given in the theory section (see [16.2.4\)](#page-264-1). The program will use the generated cut points if the cut_points parameter is not specified here. If it is specified with valid attributes and values, these preset cut points will be used instead of the cut points values as given in the theory section (see [16.2.4\)](#page-264-1).

Example: All of the following are valid RELTEST analysis statements :

```
reltest
reltest, out=test { # generate summary output file named "test.sum"
                    # and nuclear family output file named "test.fam"
   nucfam = true
   pair_type = sib # do calculations for putative sibs
   pair_type = hsib # do calculations for putative half sibs
}
reltest {
   detailed=true
   region="Chr1"
   region="Chr2"
   region="Chr42" # Do analysis using only Chromosomes 1, 2, and 42
}
```
3. See [16.4.2](#page-272-0)

16.4 Program Output

RELTEST produces several output files that contain results and diagnostic information:

16.4.1 Reclassification Summary File

The reclassification summary file contains the cut point values to classify pairs and the total length of genome used in the analysis. It also provides a separate table for each putative pair type, listing pairs to be reclassified with: their individual IDs and pedigree IDs from the original data file, new class, sibling classification statistic, parent offspring classification statistic and missing genotype rate. Note: incorrect reclassification may occur if one or both members of the pair have a high rate of missing genotypes. For each putative pair type, the total number of original pairs and the total number of pairs to be reclassified are also included.

This file also provides text-based histograms of the sibling classification statistic and the parent offspring classification statistic for each putative pair type included in the analyses. The minimum and maximum values of these statistics are also included.

Example:

robust (L2) variance : 0.5 Parent/Offspring Classification Statistics(Yj*) robust (L2) mean : -0.502698 robust (L2) variance : 0.5 Average Yj of Pairs Reclassified as Full Sibs : 1.18377 Standard Error : 0.0477349
95% Confidence Interval : 1.0883 to 1.27924 95% Confidence Interval ! WARNING : THE MEAN OF THE SIB-PAIR DISTRIBUTION DIFFERS SIGNIFICANTLY FROM ZERO. YOU MAY HAVE SUBSTANTIAL DATA ERROR OR MISSPECIFICATION OF PARAMETERS SUCH AS ALLELE FREQUENCIES. === PUTATIVE FULL SIB PAIRS TO BE RECLASSIFIED : reclassified pid pair pair type Yj Yj* missing data --- 4 4/3 HALF SIB -2.8302 -1.6862 1% / 2% 45 4/3 HALF SIB -3.0415 -0.6532 5% / 3% 58 6/5 HALF SIB -3.1239 -1.6834 4% / 3% 60 5/3 HALF SIB -2.7622 -1.0231 4% / 3% 66 31/30 HALF SIB -2.7980 -1.1299 5% / 5% 66 16/12 MZTWINS 8.3388 7.1082 4% / 30% 118 4/3 HALF SIB -2.2236 -1.6296 5% / 4% 159 5/3 HALF SIB -4.2079 0.0942 8% / 8% -- total putative pairs : 342 total pairs to be reclassified : 8 . . . == == == == HISTOGRAM OF SIBLING CLASSIFICATION STATISTIC (Yj) ==
== FOR PUTATIVE PAIRS == FOR PUTATIVE PAIRS == == == == putative pair type : FULL SIB == == maximum Yj : 8.33881 == == minimum Yj : -4.20787 == == bin size : 0.25 == == == == Interval count (one * is equal to 1 or 2 pairs.) == -4.22 to -3.97 1 * -3.97 to -3.72 0 -3.72 to -3.47 0 -3.47 to -3.22 0 -3.22 to -2.97 2 $*$ -2.97 to -2.72 3 ** -2.72 to -2.47 0
 -2.47 to -2.22 1 *
 -2.22 to -1.97 0 -2.47 to -2.22 1 $*$ -2.22 to -1.97 0 -1.97 to -1.72 0 -1.72 to -1.47
 -1.47 to -1.22

1 * -1.47 to -1.22
 -1.22 to -0.97
 -0.97 to -0.72
 $2 *$ -1.22 to -0.97 1 * -0.97 to -0.72 2 * -0.72 to -0.47 2 * -0.47 to -0.22 8 **** -0.22 to 0.03 13 ******* 0.03 to 0.28 22 *********** 0.28 to 0.53 27 ************** 0.53 to 0.78 26 ************* 0.78 to 1.03 42 ********************* 1.03 to 1.28 43 ********************** 1.28 to 1.53 34 ***************** 1.53 to 1.78 36 ****************** 1.78 to 2.03 21 ***********

16.4.2 Sibling in Nuclear Family Information File

The sibling-in-nuclear-family information file contains information about all sib pairs in nuclear families in which at least one sib pair should be reclassified. This file provides heuristic information intended to aid understanding the statistical distribution related to pairs that should be reclassified.

Example:

. . .

16.4.3 Detailed Pair Information File

This file provides a table of the Y_j and Y_j^* values for all pairs used in the analysis for each putative pair type.

Example:

Chapter 17

SEGREG

SEGREG is a very general program that can be used for, among other things, commingling analysis, segregation analysis and to produce penetrance files for model-based linkage analysis (for use in the programs LODLINK and MLOD - the latter for autosomal linkage only). The most significant improvements over the programs REGC, REGD and REGTL of the early versions of S.A.G.E., all of which are now subsumed in SEGREG, are as follows:

- 1. It is no longer necessary to provide initial parameter estimates (but these can be provided if desired).
- 2. It is no longer necessary (or possible) to specify parameters that control the maximizing process.
- 3. Several related analyses can be automatically performed in a single run.
- 4. When a transformation of the data is performed, all location parameter estimates refer to the data on their original scale of measurement - but parameter estimates of dispersion still refer to the transformed variable.
- 5. All covariates are initially centered, and the centering (average) values are given as part of the output.

17.1 Limitations

As with most S.A.G.E. programs, SEGREG cannot currently be used in the presence of pedigree loops.

Further, if the sample size is small relative to the number of parameters being estimated, the likelihood may have multiple maxima. There is no guarantee that in such a situation the maximum found and reported by the program is also the global maximum, though this is very likely. Also, as with the previous segregation analysis programs, situations can occur in which it is not numerically possible to calculate the variance-covariance matrix of the estimates.

All covariates are centered prior to entering any analysis and the means used to do this are displayed at the beginning of the output. However, these means are based on all the data available in the pedigree data file, whereas any particular analysis uses only those records informative for all variables relevant to the analysis. Thus the centering will only be exact when none of the covariates used in the analysis have missing values.

Whenever a model is maximized, the corresponding ln likelihood and -(twice the ln likelihood) are given for the estimated model. However, these values differ from the true values by a constant that is the same for all analyses performed in the same SEGREG run, but might differ, for the same data, in separate SEREG runs.

Although X-linked traits can now be analyzed under the assumption of equal allele frequencies in the two sexes, be aware that the main purpose of including X-linkage in SEGREG is to produce an X-linkage model to be used in LODLINK; for this purpose SEGREG will appropriately estimate all the parameters of a given X-linked model, but when X-linkage is specified in the transmission sub-block. the distribution of the test statistics produced at the end of a SEGREG run are not all as stated in the theory section for autosomal inheritance.

17.2 Theory

The segregation of a possible major locus is allowed for by letting one or more parameters depend on an unobserved (latent) qualitative factor $u = AA$, AB or BB . Following Go et al. (1978), we call u an individual's *type* . In this context, type is best defined in terms of the expected distribution of an individual's offspring. Two individuals have the same type if and only if the expected phenotypic distributions of their offspring by a mate of a given type are identical, and this is true for every type of mate. The same concept, but not with this definition, was denoted *ousiotype* by Cannings et al. (1978). Genotypes are the special case of types, or ousiotypes, that transmit to offspring in Mendelian fashion.

Thus we use the term *type* to allow for many kinds of discrete transmission, whether Mendelian or not. When there is no transmission from one generation to the next, the model can include the existence of only one type as defined above. In this situation, it will nevertheless be convenient to refer to several types, each with its own phenotypic distribution, but it must be understood that the model then essentially allows for only a single type, the corresponding phenotypic distribution being a mixture distribution. The incorporation of types introduces two sets of parameters, type frequencies^{[1](#page-275-0)} and transmission^{[2](#page-275-1)} parameters. The population frequencies of the types are designated ψ _{*u*}, for $u = AA$, AB, BB, and satisfy the condition:

$$
\sum_u \psi_u = 1.
$$

However, for hemozygous males, there are only two types, $u = A$, B, and $\psi_A + \psi_B = 1$.

If the type frequencies are in Hardy-Weinberg equilibrium proportions, then they are defined in terms of q_A = frequency of (component allele) A. Thus:

$$
\psi_{AA} = q_A^2
$$
; $\psi_{AB} = 2q_A(1 - q_A)$; $\psi_{BB} = (1 - q_A)^2$,

$$
\psi_A=q_A;\,\psi_B=(1-q_A).
$$

¹We use the word *frequencies* in the sense used by geneticists, i.e., *relative frequencies* that sum to 1.

²SEGREG uses the terms *transmission probability* and *transition probability* as defined by Elston and Stewart (1971).

Each transmission parameter τ_u is the probability that a parent of type u transmits allele (more generally, *component*) A to offspring, for u = AA, AB, BB. Mendelian transmission corresponds to the case in which $\tau_{AA} = 1$, $\tau_{AB} = 0.5$, and $\tau_{BB} = 0$; or $\tau_A = 1$ and $\tau_B = 0$ to only female offspring. These parameters give rise to transition^{[3](#page-276-0)} probabilities. The transition probability $Pr(u|u_F, u_M)$ is the probability that parents of types u_F (for the father) and u_M (for the mother) produce offspring of type u. Transition probabilities assume random mating and hence are determined by the transmission probabilities as follows:

$$
Pr(AA|u_F, u_M) = \tau_{u_F} \tau_{u_M}, \nPr(AB|u_F, u_M) = \tau_{u_F} (1 - \tau_{u_M}) + \tau_{u_M} (1 - \tau_{u_F}), \nPr(BB|u_F, u_M) = (1 - \tau_{u_F}) (1 - \tau_{u_M}), \nPr(A|u_F, u_M) = \tau_{u_M}, \nPr(B|u_F, u_M) = 1 - \tau_{u_M}.
$$

However, in the case that there is homogeneity of the phenotypic distributions between generations and no parent-offspring transmission of type, we define Pr(u | u_F , u_M)= τ_u , for u = AA, AB, BB, A, B. In order to have homogeneity across generations when there is parent-offspring transmission of type,

- 1. the type frequencies must be in Hardy-Weinberg equilibrium proportions, and
- 2. τ_{AB} must be a specific function of τ_{AA} , τ_{BB} and the allele frequency q_A for autosomal transmission, and other constraints are necessary for X-linked transmission (Demenais and Elston, 1981).

Details of the pedigree likelihoods that are calculated, on the assumption of random mating, are given below. It should be noted that singletons (unrelated individuals) may be included in the data. Although SEGREG counts and treats them separately for convenience, they are in fact simply one-person pedigrees and, as such, require no special treatment in the model. However, note that these singletons are not considered to be founders. Estimation is performed by maximum likelihood and standard errors are obtained by numerical double differentiation of the log likelihood surface. The output contains the overall ln(likelihood), -2ln(likelihood) and Akaike's *A* information criterion $(AIC)^4$ $(AIC)^4$ for each of the models that has been maximized in a run. When the model consists of two or three types, a table is produced indicating the respective likelihood ratio statistic for each type.

Transmission models are compared and p-values quoted according to the asymptotic distribution of the likelihood ratio for autosomal transmission (some of which are not appropriate for X-linked transmission) as shown in the table below (Self and Liang, 1987). In this table, the following abbreviations are used to describe the models:

 3 ditto

⁴Contrary to popular belief, the acronym AIC stands for the *A Information Criterion* defined by Akaike, and not *Akaike's Information Criterion* .

See also note 2 of the transmission sub-block $(17.3.2.12)$.

In the description of the models from here on, X-linkage is largely ignored. It is assumed that β_{AA} = β_A and $\beta_{BB} = \beta_B$ and, when the trait is indicated as being X-linked, the segreg.typ file (see [17.4\)](#page-329-0) is approperiate for a trait/marker in LODLINK.

17.2.1 Segregation Models

Certain aspects of the models available in SEGREG are common to all traits and models, and are described here. Later sections describe the aspects that are specific to regressive models for quantitative traits, regressive multivariate logistic models for binary traits, the finite polygenic mixed model, and models for binary traits with variable age of onset. Because SEGREG analyzes all constituent pedigrees as being independent, in the rest of this chapter we simply use the word " pedigree" to mean "consituent pedigree".

17.2.1.1 Ascertainment: Conditioning on a Subset

Instead of being sampled at random, a pedigree may be included in the analysis because one or more members of the pedigree have particular trait values or are in a certain *sampling frame*^{[5](#page-278-0)}. It may be desireable to condition the likelihood on the phenotypes of these individuals or, more generally, on the phenotypes and/or structure of any prespecified subset C of the pedigree. This *conditioned subset* may be

 5 The pedigree sampling frame can include pedigree members for whom the trait value is missing, in which case calculation will proceed as in Ginsberg et al. (2003). This is not advisable, however. A better strategy is to replace these missing values by the average of the observed (i.e., non-missing) trait values.

- 1. the set of founders (members of the pedigree whose parents are not included in the pedigree),
- 2. the set of pedigree members in the pedigree proband sampling frame, or
- 3. the union of these two sets.

Currently, no model is assumed for the ascertainment and, for results to be correct, the observed pedigree must contain all members of the pedigree proband sampling frame. This subsumes both simplex and multiplex single ascertainment (see Elston and Bonney, 1986) as special cases. In the case of simple single ascertainment, the pedigree proband sampling frame for each pedigree comprises only the proband. See Ginsberg et al (2006) for a discussion of what is meant by "pedigree", "correct results" and "pedigree proband sampling frame".

If no conditioned subset is indicated for a particular pedigree (either explicitly as a user-specified set or implicitly as the founders), random sampling is assumed for that pedigree. In general, the likelihood for a randomly sampled pedigree (L) is divided by a correction L_C , defined in one of three possible ways.

1. Random Sampling

In this case, no correction is necessary, so C is empty and we define $L_C = 1$.

2. Conditioning on Actual Phenotypes

In this case, the likelihood is conditioned on the available phenotype of each member of the conditioned subset. The correction L_C is then taken to be L computed as though all individuals not in C are missing.

3. Conditioning on Phenotypes Being Above or Below a Threshold Value

In this case, the likelihood is conditioned, for each member of the conditioned subset for whom a phenotype is available, on that member's phenotype being at least as large as a threshold T_U , or at most as large as a threshold T_L .

17.2.1.2 Type Probabilities and Penetrance Functions

Given a model with established parameter values, we estimate the posterior probability of each possible type for every individual, conditional on all the sample data. We define the following terms:

- $L(\bullet)$ is a likelihood
- *S* is the set of all sampled data in the pedigree and
- *uⁱ* is the type of individual *i.*

Then the posterior probability for a given individual is computed (using maximum likelihood estimates of unknown parameters) as:

$$
L(u_i|S) = \frac{L(u_i, S)}{L(S)}.
$$

Note that the denominator is the likelihood L computed for the whole pedigree to which *i* belongs.

If *uⁱ* is a genotype, SEGREG can also prepare files of penetrance functions that can be used as input into LODLINK and MLOD (the latter for autosomal linkage only) using maximum likelihood estimates of all unknown parameters. These are of the form $Pr(t_i|u_i)$, where t_i is the analysis trait (see [17.2.2.2\)](#page-281-0).

17.2.2 Regressive Models for Quantitative Traits

Regressive models (Bonney, 1984; 1998) are those models in which distributions over pedigrees are specified by conditioning each individual's trait value on those of antecedent individuals. For a quantitative trait they assume (possibly after transformation) multivariate normality across pedigree members of the underlying individual residuals from the type means. Two classes of regressive models for quantitative traits are implemented in SEGREG. Class A models assume that sibling subtypes are dependent only because of common parentage, while class D models assume that the sibling correlations are equal, but not necessarily due to common parentage alone. For a quantitative trait, SEGREG assumes a model that is a close approximation to multivariate normal for the underlying individual residuals. The approximation used is a generalization of approximation 6 in Demenais et al (1990).

The following correlations among the residuals from the type means can be allowed in all the models: ρ*FM* for father-mother (spouse), ρ*MO* for mother-offspring, ρ*FO* for father-offspring, and ρ_{SS} for any two siblings (in a class D model). A class A model also includes, indirectly, a sibling correlation ρ_{SS} that satisfies the condition

$$
\rho_{SS} = \frac{\rho_{MO}^2 + \rho_{FO}^2 - 2\rho_{FM}\rho_{MO}\rho_{FO}}{1 - \rho_{FM}^2}.
$$

The residual correlations between half siblings are assumed to be zero, conditional on the common parent. Missing values are handled according to the formulas in Bonney (1984, 1998), with the result that, for example, the residual grandparent-grandchild correlation is assumed to be zero if the intervening parent has a missing phenotype.

In the correlation structure indicated above, the means and variances of the underlying normal distribution can be dependent on covariates. All covariates are centered prior to inclusion in the likelihood.

The correlation parameters (ρs) are the correlations of the residual multivariate normal distribution. Thus the inference of a major gene can be made allowing for the cumulative effect, assumed to be multivariate normally distributed for the transformed trait, of various factors (such as polygenes, cultural, and other environmental factors) that are not separately distinguished.

17.2.2.1 Composite Trait

The trait to be analyzed may be a single variate, the *main trait* ($y = y^*$) or a linear function of the main trait (with coefficient 1) and *p* covariates (with coefficients κ_i):

$$
y = y^* + \kappa_1 x_1 + \kappa_2 x_2 + \ldots + \kappa_{p_k} x_p,
$$

where the parameters κ*ⁱ* may be estimated.

17.2.2.2 Transformation of the Trait

The trait *y*, however composed, may be transformed by one of two transformations. For commingling analysis and segregation analysis, the first (Box and Cox) transformation is recommended.

The first possible transformation is:

$$
t = h(y) = \begin{cases} \frac{(y + \lambda_2)^{\lambda_1} - 1}{\lambda_1(y_{G1})^{(\lambda_1 - 1)}} & \text{if } \lambda_1 \neq 0, \\ y_{G1} \ln(y + \lambda_2) & \text{if } \lambda_1 = 0 \end{cases}
$$

where

$$
y_{G1} = \left[\prod_{i=1}^{N} (y_i + \lambda_2)\right]^{\frac{1}{N}}
$$

and $N =$ number of individuals in the full data set (possibly including more than one pedigree) with complete trait and covariate values (nothing missing). This is the standardized Box and Cox (1964) transformation with power parameter λ_1 and shift parameter λ_2 .

The second possible transformation is is the standardized generalized modulus power transformation (George and Elston, 1988) with power parameter λ_1 and shift parameter λ_2 (see [5.2.2\)](#page-97-0).

We call the transformed trait *t* the *analysis trait*. When a transformation is applied it is applied to *both sides* of the regression equation (Carroll and Ruppert, 1984), so that all location parameters are median unbiased on the original scale of measurement.

17.2.2.3 Likelihood for a Randomly Sampled Pedigree

Let the pedigree contain n individuals $(i = 1, ..., n)$ on each of whom we observe a value of the analysis trait. An individual's analysis trait is considered missing if the value of any variate for that individual, required for calculating the likelihood, is unknown. For individual i, let

 t_i = analysis trait value of i

 x_{ij} = j-th covariate value of i

 u_i = type of i

- S_i = spouse of i
- M_i = mother of i
- F_i = father of i
- B_{ij} = j-th observed elder sibling of i

 n_{iB} = number of observed elder siblings of i.

We let the expected value of *t* conditional on type *u* be

$$
\theta_u(i) = h(\beta_u + \xi_1 x_{i1} + \xi_2 x_{i2} + ... + \xi_{p_{\xi}} x_{ip_{\xi}})
$$

and the variance of *t* conditional on type *u* be

$$
\eta_u^2(i) = \sigma_u^2 + \varsigma_1 x_{i1} + \varsigma_2 x_{i2} + \ldots + \varsigma_{\varsigma} x_{ip_{\varsigma}}
$$

Note we assume $\sigma_A^2 = \sigma_{AA}^2$ and $\sigma_B^2 = \sigma_{BB}^2$; any sex difference must be allowed for by adding a sex covariate.

Because the expected value of *t* conditional on type *u* undergoes the same transformation as is used to produce *t* ("transformation of both sides", see Carroll & Ruppert, 1984), the estimates of parameters in this conditional expectation are median unbiased on the same scale of measurement as the original untransformed data. However, the residual variance that is calculated, and all the covariate coefficients pertaining to it, are on the scale of the analysis trait. Further general quantities that apply to regressive models are defined as follows:

$$
\alpha_{iS} = \left\{ \begin{array}{ll} \rho_{FM} & if specific space of i is observed, \\ 0 & otherwise, \end{array} \right.
$$

$$
\alpha_{iM} = \begin{cases}\n\frac{\rho_{MO} - \rho_{FO}\rho_{FM}}{1 - \rho_{FM}^2} & \text{if both parents of i are observed,} \\
\rho_{MO} & \text{if mother, but not father, of i is observed,} \\
0 & \text{if mother of i is not observed,}\n\end{cases}
$$

$$
\alpha_{iF} = \begin{cases} \begin{array}{c} \frac{\rho_{F0} - \rho_{MO} \rho_{FM}}{1 - \rho_{FM}^2} & \text{if both parents of i are observed,} \\ \rho_{FO} & \text{if father, but not mother, of i is observed,} \\ 0 & \text{if father of i is not observed,} \end{array} \end{cases}
$$

$$
\delta_i = \alpha_{iM}\rho_{MO} + \alpha_{iF}\rho_{FO} = \begin{cases}\n\rho_{SS} & if both parents of i are observed, \\
\rho_{MO}^2 & if mother, but not father, of i is observed, \\
\rho_{FO}^2 & if father, but not mother, of i is observed, \\
0 & if neither parent of i is observed,\n\end{cases}
$$

$$
\phi(z_i, w_i) = \frac{1}{\sqrt{2\pi w_i}} \exp[-z_i^2/(2w_i)],
$$

where the arguments *zⁱ* and *wⁱ* are defined differently for each of the model classes. For a class A model, the arguments of the normal density function are defined in SEGREG as

$$
z_i = t_i - \theta_u(i) - b_{iS} V_{iS_i}(t_{S_i} - \theta_u(S_i)) - b_{iM} V_{iM_i}(t_{M_i} - \theta_u(M_i)) - b_{iF} V_{iF_i}(t_{F_i} - \theta_u(F_i))
$$

and

$$
w_i = \eta_u^2(i)(1 - b_{iS}\rho_{FM} - b_{iM}\rho_{MO} - b_{iF}\rho_{FO}),
$$

where

$$
V_{ij} = \eta_u(i)/\eta_u(j)
$$

 $b_{iS} = \alpha_{iS}$,

$$
b_{iM} = \alpha_{iM} \left(\frac{1 - \rho_{SS}}{1 - \rho_{SS} + n_{iB}(\rho_{SS} - \delta_i)} \right)
$$

$$
b_{iF} = \alpha_{iF} \left(\frac{1 - \rho_{SS}}{1 - \rho_{SS} + n_{iB}(\rho_{SS} - \delta_i)} \right),
$$

with

$$
\rho_{SS} = \frac{\rho_{MO}^2 + \rho_{FO}^2 - 2\rho_{MOFOFM}}{1 - \rho_{FM}^2}.
$$

For a class D model, the arguments of the normal density function are defined as:

$$
z_i = t_i - \theta_u(i) - b_{iS}V_{iS_i}(t_{S_i} - \theta_u(S_i)) - b_{iM}V_{iM_i}(t_{M_i} - \theta_u(M_i))
$$

- $b_{iF}V_{iF_i}(t_{F_i} - \theta_u(F_i)) - b_{iB}\sum_{j=1}^{n_{iB}}\hat{V}_{iB_{ij}}(t_{B_{ij}} - \hat{\mu}_{B_{ij}}),$

and

$$
w_i = \eta_u^2(i)(1 - b_{iS}\rho_{FM} - b_{iM}\rho_{MO} - b_{iF}\rho_{FO} - n_{iB}b_{iB}\rho_{SS}),
$$

where

$$
\hat{\mu}_j = \sum_{u_j} \theta_u(j) f_{uj} / \sum_{u_j} f_{uj}
$$
\n
$$
f_{uj} = Pr(u_j | u_{F_j}, u_{M_j})_j - \theta_u(j))^2 / (2 \eta_u^2(j)) \} / \eta_u(j),
$$
\n
$$
\hat{\sigma}_j^2 = \sum_u f_{uj} \sigma_u^2 / \sum_u f_{uj},
$$
\n
$$
\hat{V}_{ij} = \sum_{u_j} f_{uj} / \sum_{u_j} f_{uj} \eta_u^2(j)
$$
\n
$$
b_{iS} = \alpha_{iS}
$$
\n
$$
b_{iM} = \alpha_{iM} \left(\frac{1 - \rho_{SS}}{1 - \rho_{SS} + n_{iB}(\rho_{SS} - \delta_i)}, \right)
$$

To indicate all the potential variables in $\phi(z_i, w_i)$, except covariates, denote it

$$
Pr(t_i|u_i,u_S,u_M,u_F,t_{S_i},t_{M_i},t_{F_i},t_{B_{il}},...,t_{B_{in_{iB}}}).
$$

(This quantity is a conditional phenotypic density function, sometimes referred to as a penetrance function.)

Using the components defined above, let

$$
p_i(u_i, u_{M_i}, u_{F_i}) = \begin{cases} Pr(u_i|u_{F_i}, u_{M_i}) & if the parents of i are included in the pedigree, \\ \psi_i & otherwise, \end{cases}
$$

and

 $H_i(u_i, u_{S_i}, u_{M_i}, u_{F_i}, t_i, t_{S_i}, t_{M_i}, t_{F_i}, t_{B_{il}}, ..., t_{B_{in_{iB}}})$

$$
= \begin{cases} p_i(u_i, u_{M_i}, u_{F_i}) & if i missing, \\ p_i(u_i, u_{M_i}, u_{F_i}) Pr(t_i|u_i, u_{S_i}, u_{M_i}, u_{F_i}, t_{S_i}, t_{M_i}, t_{F_i}, t_{B_{i1}}, ..., t_{B_{in_{iB}}}) & otherwise \end{cases}
$$

Then under random mating the likelihood for a randomly sampled pedigree is

$$
L = \left[\sum_{u_1} \dots \sum_{u_n} \prod_{i=1}^n H_i(u_i, u_{S_i}, u_{M_i}, u_{F_i}, t_i, t_{S_i}, t_{M_i}, t_{F_i}, t_{B_{il}}, \dots, t_{B_{in_{iB}}})\right].
$$

17.2.2.4 Allowing for Ascertainment

Ascertainment is allowed for as indicated in [17.2.1.1.](#page-278-1) In order to condition on traits being at least as large as T_U or at most as large as T_L , the correction L_C is taken to be the likelihood defined in [17.2.1.1](#page-278-1) computed as though all individuals not in the prespecified subset C are missing, but with $Pr(t_i|.)$, for each individual i in C replaced by

$$
\int\limits_{U}^{\infty} Pr(t|.)dt = \Phi(-z_{iU}/\sqrt{w_i}), or \int\limits_{-\infty}^{T_L} Pr(t|.)dt = \Phi(z_{iL}/\sqrt{w_i}),
$$

where z_{iU} or z_{iL} is identical to z_i with $h(T_U)$ or $h(T_L)$, respectively, substituted for t_i . However, z_i is always left unchanged for any founders not included in the proband sampling frame.

17.2.3 Regressive Multivariate Logistic Models for Binary Traits

The multivariate logistic model for a binary trait was described by Karunaratne and Elston (1998) for nuclear family data. It is implemented in SEGREG for pedigree data by making the regressive model assumption that, conditional on the trait and major type of any individual who belongs to two nuclear families, the likelihoods for those two nuclear families are independent. In this model, unlike in Bonney's (1986) multiple logistic model, the marginal probability that any pedigree member has a particular trait is the same for all members who have the same values of any covariates in the model. This marginal probability, which we call susceptibility, is given by the cumulative logistic function

$$
\gamma = \frac{e^{\theta(i)t_i}}{1 + e^{\theta(i)}},
$$

where t_i , the analysis trait of the i-th individual, is 1 for an affected individual and 0 for an unaffected individual; and $\theta(i)$, the logit of the susceptibility for the i-th individual, can depend on both major type (u) and covariate values x_{i1} , x_{i2} , ..., x_{ip} :

$$
\theta_u(i) = \beta_u + \xi_1 x_{i1} + \ldots + \xi p x_{ip}.
$$

Composite trait transformation is not relevant for a binary trait; nor is a Class A model possible.

Nuclear family residual association parameters, analogous to the correlation parameters in regressive models for quantitative traits, are incorporated into the model. These are denoted in [17.2.4](#page-286-0) below as δ_{FM} for father-mother (spouse), δ_{MO} for mother-offspring, δ_{FO} for father-offspring, and δ_{SS} for any two siblings. In the case of the multivariate logistic distribution these association parameters correspond to second-order correlations; it is assumed that all higher order correlations are zero. The actual correlations are calculated from these associations measures for specific logit values [see Karunaratne and Elston (1988)]. Note that, because all covariate values are centered, the logit values are at the sample average value of all covariates.

For a binary trait, information about the population prevalence of the trait (for a binary trait with variable age of onset, the probability of having been affected since birth) can be incorporated into the likelihood as an independent factor. This is done by specifying that a sample of N independent individuals have been observed, of whom R have been affected for given values of the covariates (and/or up to a specified age), and this may be repeated for different sets of covariate values (The corresponding factor(s) in the likelihood are not shown in the next section). Similarly, the program can output the prevalence, for given sets of covariate values (and/or up to a specified age), estimated from the model using the maximum likelihood estimates of all parameters.

17.2.3.1 Likelihood for a Randomly Sampled Nuclear Family

Let t_F , t_M and t_i be the traits of the father, mother and i-th child, i = 1, 2, ..., n and u_F , u_M and u_i be the types of the father, mother and i-th child. Then the likelihood for a nuclear family is

$$
\sum_{u_F} \sum_{u_M} \sum_{u_1} \cdots \sum_{u_n} Pr(u_F) Pr(u_M | u_F) \prod_{i=1}^n Pr(u_i | u_F, u_M) L(t_F, t_M, t_1, \ldots, t_n | u_1, \ldots, u_n),
$$

where $Pr(u_F)Pr(u_M|u_F)$ is the joint probability of types u_F and u_M in the population; $Pr(u_i|u_F, u_M)$ is the probability that a sib has type u_i , given the parents' types are u_F and u_M ; and the penetrance function $L(t_F, t_M, t_1, \ldots, t_n | u_F, u_M, u_1, \ldots, u_n)$ is given by

$$
\prod_{i=F,M,1}^{n} \frac{e^{\theta_{u}(i)t_{i}}}{1+e^{\theta_{u}(i)}} \left\{ 1+\delta_{FO} \left(1 - \frac{e^{\theta_{u}(F)t_{F}}}{1+e^{\theta_{u}(F)}} \right) \sum_{i=1}^{n} (-1)^{t_{F}+t_{i}} \left(1 - \frac{e^{\theta_{u}(i)t_{i}}}{1+e^{\theta_{u}(i)}} \right) \right. \\
\left. + \delta_{MO} \left(1 - \frac{e^{\theta_{u}(M)t_{M}}}{1+e^{\theta_{M}(u)}} \right) \sum_{i=1}^{n} (-1)^{t_{M}+t_{i}} \left(1 - \frac{e^{\theta_{u}(i)t_{i}}}{1+e^{\theta_{u}(i)}} \right) \\
+ \delta_{SS} \sum_{1 \leq i < j \leq n} (-1)^{t_{i}+t_{j}} \left(1 - \frac{e^{\theta_{u}(i)t_{i}}}{1+e^{\theta_{u}(i)}} \right) \left(1 - \frac{e^{\theta_{u}(j)t_{j}}}{1+e^{\theta_{u}(j)}} \right) \left. \left(1 - \frac{e^{\theta_{u}(j)t_{j}}}{1+e^{\theta_{u}(j)}} \right) \right\}.
$$

17.2.4 Finite Polygenic Mixed Model

The finite polygenic mixed model (Fernando et al, 1994; Lange, 1997) can be used for either quantitative or binary traits, the only difference being in the particular penetrance function used. It can also be used for binary traits with variable age of onset.

In addition to type (AA, AB or BB), we assume the presence of k diallelic polygenic loci in the model. The alleles at each such locus are a and b, with effects α and β , and frequencies p and 1 p (the default value of p is 0.5). The polygenic effect is the sum of the effects of alleles at all k loci. Thus, if a pedigree member has *v* a alleles and $(2k - v)$ b alleles, then the polygenic effect is

$$
\mu_v = v\alpha + n(2k - v)\beta,
$$

where *v* is called the polygenic number, and α and β are chosen to make the mean polygenic effect zero. It follows that

$$
\mu_v = \frac{v - 2pk}{1 - p} \sqrt{\frac{\sigma_v^2(1 - p)}{2pk}},
$$

where σ_v^2 is the variance of the polygenic effect.

We assume that, conditional on the polygenic numbers of two parents, the polygenic number of any pedigree member is independent of the polygenic numbers of all other pedigree members. This allows us to use the Elston-Stewart (1971) algorithm summing over the $2k +1$ possible genetic numbers times the three possible types. Although this is not strictly consistent with Mendelian inheritance, it leads to a conditional correlation of zero between the polygenic numbers of any two pedigree members.

It is possible to analyze a composite trait and to transform the trait in the case of a quantitative trait. As for regressive models for quantitative traits, the type mean and/or variances can depend on covariates. For a quantitative trait, let t_i be the analysis trait for individual i , with expectation conditional on type *u*:

$$
\theta_u(i) = h(\beta_u + \xi_1 x_{i1} + \xi_2 x_{i2} + ... + \xi_{p_{\xi}} x_{ip_{\xi}})
$$

and let the variance of *t* conditional on type *u* be

$$
\eta_u^2(i) = \sigma_u^2 + \zeta_1 x_{i1} + \zeta_2 x_{i2} + \ldots + \zeta_s x_{ip_s}.
$$

Then in the finite polygenic mixed model we define the penetrance function for a quantitative trait to be

$$
Pr(t_i|u_i,v_i) = \varphi(t_i - \theta_u(i) + \mu_{v_i}, \sigma_i^2),
$$

with polygenic variance equal to the variance of μ_{v_i} .

In the case of a binary trait, we define the penetrance function to be the cumulative logistic function

$$
Pr(t_i|u_i,v_i)=\frac{e^{\theta_u(i)}}{1+e^{\theta_u(i)}},
$$

where, conditional on type *u* , we have the logit

$$
\theta_u(i) = \beta_u + \mu_{v_i} + \xi_1 x_{i1} + \xi_2 x_{i2} + \ldots + \xi_{p_{\xi}} x_{ip_{\xi}}.
$$
17.2.4.1 Likelihood for a Randomly Sampled Pedigree

Using the penetrance functions defined above, and letting

 $P_i(u_i, u_{Mi}, u_{Fi}, v_i, v_{Mi}, v_{Fi}) =$

 $\begin{cases} Pr(u_i, u_{Mi}, u_{Fi}, v_i, v_{Mi}, v_{Fi}) & \text{if the parents of } i \text{ are included in the pedigree} \\ \psi_i & \text{otherwise} \end{cases}$

and

$$
H_i(u_i, v_i, z_i) = \begin{cases} Pr(u_i, u_{M_i}, u_{F_i}, v_i, v_{M_i}, v_{F_i}) & if i is missing, \\ Pr(u_i, u_{M_i}, u_{F_i}, v_i, v_{M_i}, v_{F_i}) Pr(t_i|u_i v_i) & otherwise \end{cases}
$$

under random mating the likelihood for a randomly sampled pedigree is

$$
L=\sum_{u_1}\ldots\sum_{u_n}\sum_{v_1}\ldots\sum_{v_n}\prod_{i=1}^nH_i(u_i,v_i,z_i).
$$

17.2.5 Binary Traits with Variable Age of Onset

When using the command line, SEGREG requires specification of the fpmm for a binary trait with variable age of onset, but it is then possible to specify 0 polygenic loci. When using the GUI it is possible to chose "binary trait whith variable age of onset" without specifying the fpmm.

In general terms, letting a be age of onset and a' the age at examination (for an unaffected person, the last age at which a person is known to be so), the penetrance functions are:

- $\gamma(f(a))$ for an affected person with known age of onset *a*,
- $\gamma(F(a'))$ for an affected person with unknown age of onset, age at examination a' , and
- $1 \gamma(F(a'))$ for an unaffected person with age at examination *a'*,

where γ is the susceptibility and $f(a)$ is the age of onset density with cumulative distribution $F(a')$.

The mean and variance of f , and the susceptibility γ , can each be made dependent on covariates and/or type, in the same way as for a quantitative analysis trait and a binary trait, respectively. However, age of onset is assumed to follow a logistic density function rather than a normal density function. Letting β be a baseline parameter and α the age coefficient, the density and cumulative distribution functions are:

$$
f(a) = \frac{\alpha e^{\beta + \alpha a}}{(1 + e^{\beta + \alpha a})^2}
$$

$$
F(a') = \frac{e^{\beta + \alpha a'}}{1 + e^{\beta + \alpha a'}} = [1 + e^{-(\beta + \alpha a')}]^{-1}
$$

For this distribution, the mean = $-\frac{\beta}{\alpha}$ $\frac{\beta}{\alpha}$, and the variance = $\frac{\pi^2}{3\alpha}$ $rac{\pi^2}{3\alpha^2}$.

The mean and variance of the age of onset distribution can each depend linearly on covariates, and transformation of "both sides" is possible. Using the logistic distribution has the advantage that the parameters α and β can be interpreted as increases in log odds in the susceptible population (in the whole population if $\gamma = 1$). However, the variance of the logistic distribution depends on the mean, and so it is not permissible for the mean and variance to depend on the same covariate.

The susceptibility γ is modeled by a cumulative logistic, in the same way as a binary trait is modeled. In order to avoid confounding among the parameters, there are restrictions on how age of onset and susceptibility depend on type and polygenic number in the case of the finite polygenic mixed model. The following six possibilities are permissible:

- 1. Age of onset depends on major genotype alone, susceptibility depends on neither major genotype nor polygenic number
- 2. Age of onset depends on both major genotype and polygenic number, susceptibility depends on neither
- 3. Age of onset depends on major genotype alone, susceptibility depends on polygenic number alone
- 4. Susceptibility depends on major genotype alone, age on onset depends on neither
- 5. Susceptibility depends on both major genotype and polygenic number, age of onset depends on neither
- 6. Susceptibility depends on major genotype, age of onset depends on polygenic number alone

As for a binary trait without variable age of onset, information about prevalence (probability of having been affected since birth) can be incorporated into the likelihood, or estimated from the model fitted.

17.3 Program Input

17.3.1 Running segreg

A typical run of the SEGREG program may use flags to identify the file types like the following:

>segreg -p data.par -d data.ped

or, rely on a set file order like the following:

>segreg data.par data.ped

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file.

17.3.2 The segreg Block

A segreg block in the parameter file sets the options on how to perform an analysis using SEGREG. The following table shows the syntax for a segreg parameter which starts the segreg block.

The following table lists the parameters and attributes that may occur in a segreg block (see note 1).

- 1. Each of the title and trait parameters is defined by its own block. Except when a binary trait with variable age of onset is being analyzed, no sub-blocks are required (a commingling analysis is automatically performed in this case). Whenever a sub-block is included, there can be required parameters.
- 2. Only required if a trait with variable age of onset is being analyzed, or a binary trait is to be analyzed as a quantitative trait.
- 3. The trait analyzed can be a linear function of the primary trait (with coefficient 1) and other

covariates whose coefficients are fixed or estimated. This linear function is called a composite trait. Without this sub-block a composite trait is not formed. All covariates are centered, the centering (average) value being included as part of the output. The covariates can be any covariate or trait (other than the primary trait) listed in the data file or created by means of a function block. Note: This sub-block is not applicable to binary traits.

- 4. This sub-block refers to means of quantitative traits. Without this sub-block, one, two and three types are fitted successively (see notes 2 and 3 following the type_mean sub-block for an interpretation of the type means).
- 5. This sub-block refers to variances of quantitative traits conditional on type. Without this sub-block, one common variance is fitted.
- 6. This sub-block refers to logits of susceptibilities (or of penetrances). Without this sub-block, one, two and three types are fitted successively (see notes 2 and 3 following the type_suscept sub-block for an interpretation of the type susceptibilities). Note that it is not possible to fit more than one type (i.e., only one type susceptibility is estimable) when either the **no_trans** or homog_no_trans option for transmission is used unless the model includes non-zero residual associations.
- 7. This sub-block indicates which covariates are to (linearly) modify the means indicated in the type_mean sub-block. Without this sub-block, no such covariates are included in the analysis. All covariates are centered, the centering (average) value being included as part of the output.
- 8. This sub-block indicates which covariates are to (linearly) modify the variances in the type_var sub-block. Without this sub-block, no such covariates are included in the analysis. All covariates are centered, the centering (average) value being included as part of the output.
- 9. This sub-block indicates which covariates are to (linearly) modify the logits of susceptibilities (or of penetrances) indicated in the type_suscept sub-block. Without this sub-block, no such covariates are included in the analysis. All covariates are centered, the centering (average) value being included as part of the output.
- 10. The values of A and D denote Bonney's class A and D regressive models, respectively. FPMM is the finite polygenic mixed model. Without this parameter, a class D regressive model is used for quantitative traits and a multivariate logistic model is used for binary traits. FPMM is the option to choose for binary traits with variable age of onset when using the command line, even if zero polygenic loci are desired.
- 11. This sub-block is not relevant for the FPMM (finite polygenic mixed model). Residual correlations are relevant for quantitative traits and residual associations are relevant for binary traits. We use the term " correlations" to cover both situations. Without this sub-block, the usual genetic mixed model assumption of no marital correlation and equal sib-sib and parent-offspring correlations is used.
- 12. Without this sub-block, the Box-Cox power parameter that provides the best fit to a normal distribution (logistic distribution for age of onset) conditional on type is estimated. An error message will be returned if any value of the analysis trait is at any time necessarily negative. When a composite trait is being analyzed this is avoided as much as possible.
- 13. Without this sub-block, it is assumed that there is no genotype correlation between spouses, and that there are Hardy-Weinberg equilibrium proportions when fitting three types.
- 14. Without this sub-block, homogeneity across generations, no transmission, and Hardy-Weinberg equilibrium proportions are assumed.
- 15. Without this sub-block, it is assumed that the pedigrees are randomly sampled.
- 16. Without this sub-block, the estimate of population prevalence (more correctly, for a binary trait with variable age of onset, the probability of having been affected since birth) is not constrained by data extraneous to the pedigree file.
- 17. Without this sub-block, the population prevalence (for a binary trait with variable age of onset, the probability of having been affected since birth) is not calculated.
- 18. Without this sub-block, the output contains the overall ln(likelihood), -2ln(likelihood) and Akaike's AIC criterion for each of the models that has been maximized in a run.
- 19. This sub-block is not applicable to binary traits, but does apply to the age-of-onset distribution of a binary trait with variable age of onset.
- 20. This sub-block is not applicable to quantitative traits.

17.3.2.1 The composite_trait Sub-Block

The following table lists the parameters and attributes that may occur in a composite_trait subblock (see note 1).

- 1. This sub-block is not relevant for a binary trait (with or without variable age of onset). A particular trait may not be specified as both a mean covariate and as a composite trait covariate.
- 2. If the fixed attribute is set to true, the attribute val must be included. If set to false and the attribute val is included, this determines the initial value of the variable to be used in the maximization process. If set to false and the attribute val is not included, then the program supplies various initial values for the maximization process.

17.3.2.2 The type_mean Sub-Block

Notes

1. This option refers to the number of types fitted to a quantitative trait. Note that if a type_mean sub-block is not included, the program successively fits one, two and three types (see note 4 of the segreg block), and the output will include results for all three types. On the other hand, if a type_mean sub-block is included without specifying an option, then only one type is fitted. This sub-block is only relevant for quantitative traits. It is relevant for the age of onset distribution of a binary trait with variable age of onset, but is not otherwise relevant for a binary trait.

2. When specified in this sub-block, the type effects are means of continuous distributions. For a binary trait with variable age of onset, they are the mean values of age of onset.

3. Denoting the three type effects β_{AA} , β_{AB} , and β_{BB} , the options correspond to:

Note: If any of the options three_add, three_inc or three_dec is specified, the output will also include the results for options **one** and **two_dom** or **two_rec**, depending on which has the higher likelihood.

For example,

type_mean { option=three_inc mean="A*", val=-1.0, fixed=false mean="BB", val=-2.0, fixed=false }

sets initial estimates $\beta_{AA} = \beta_{AB} = -1.0$ and $\beta_{BB} = -2.0$ when estimating $\beta_{AA} \leq \beta_{AB} \leq \beta_{BB}$.

4. If X-linkage is specified in the transmission sub-block (17.3.2.12), β*AA* and β*BB* are also the means of the hemizygous males.

17.3.2.3 The type_var Sub-Block

The following table lists the parameters and attributes that may occur in a type_var sub-block (see note 1).

Notes

1. This sub-block is only relevant for quantitative traits. It is relevant for the age of onset distribution of a binary trait with variable age of onset, but is not otherwise relevant for a binary trait. There can be at most one variance for each type specified in the type_mean sub-block. When used in conjunction with the logistic density function for an age of onset distribution, only one variance is possible regardless of the number of types specified in the type_mean sub-block.

2. Denoting the three variances σ_{AA}^2 , σ_{AB}^2 and σ_{BB}^2 , the six options are analogous to the first six options in the type_mean sub-block (see note 3 of the type_mean sub-block) with σ^2 replacing $β$.

For example,

```
type_var
{
   option=three
   var="AA", val=5.0, fixed=false
   var="B*", val=30.0, fixed=true
}
```
sets the initial estimate σ_{AA}^2 =5.0 and fixes values $\sigma_{BB}^2 = \sigma_{AB}^2$ =30.0, when estimating only σ_{AA}^2 .

3. If X-linkage is specified in the transmission sub-block (17.3.2.12), σ_{AA}^2 and σ_{BB}^2 are also the valiances of the hemizygous males.

17.3.2.4 The type_suscept Sub-Block

The following table lists the parameters and attributes that may occur in a type_suscept subblock.

Notes

1. This option refers to the number of types fitted to a binary trait. Note that if a type_suscept sub-block is not included, the program successively fits one, two and three

types (see note 6 of the segreg block). On the other hand, if a type_suscept sub-block is included without specifying an option, then only one type is fitted.

2. The type effects are mean logits, $θ$, of penetrances, $γ$, (of susceptibilities for a binary trait with variable age of onset).

$$
\gamma = \frac{e^{\theta}}{1 + e^{\theta}}
$$

3. Denoting the three type effects β_{AA} , β_{AB} , and β_{BB} , the options correspond to:

If option is for more than one type effect (eg., two, three, two_dom, etc.) and the value of one of them is fixed, then initial values must be specified for all susceptibilities.

For example,

```
type_suscept
{
   option=three_inc
   mean="A*", val= -2.0, fixed=false
   mean="BB", val= -1.0, fixed=false
}
```
sets initial estimates $\beta_{AA} = \beta_{AB} = -2.0$ and $\beta_{BB} = -1.0$ when estimating $\beta_{AA} \leq \beta_{AB} \leq \beta_{BB}$ in the logit $\theta_u(i)$ as described in [17.2.3.](#page-285-0)

4. If X-linkage is specified in the transmission sub-block (17.3.2.12), β*AA* and β*BB* are also the means of the hemizygous males.

17.3.2.5 The mean_cov Sub-Block

The following table lists the parameters and attributes that may occur in a mean_cov sub-block.

- 1. The default is to include no covariates in the analysis. The means indicated in the type_mean sub-block are a linear function of this covariate. A covariate specified here cannot be also specified in the composite_trait sub-block. All covariates are centered, the centering (average) value being included as part of the output. This sub-block is only relevant for quantitative traits.
- 2. It is relevant for the age of onset distribution of a binary trait with variable age of onset, but is not otherwise relevant for a binary trait. In the case of age of onset a logistic density function

is used, and so the same covariate cannot be specified to modify both the mean and the variance; nor can the same covariate be used to modify both the mean and the susceptibility.

3. The interaction attribute refers to an interaction with type; the default is to assume no interaction. If there is no interaction, we estimate β_{AA} , β_{AB} , β_{BB} (as many as are specified in the type_mean sub-block) and one overall " mean" covariate coefficient for each covariate. If there is interaction, then for this "mean" covariate we estimate an additional two interaction effects that sum to 0 if two β parameters are being fitted; and an additional three interaction effects that sum to 0 if three β parameters are being fitted.

17.3.2.6 The var_cov Sub-Block

The following table lists the parameters and attributes that may occur in a var_cov sub-block.

- 1. The default is to include no covariates in the analysis. The variances indicated in the type_var sub-block are a linear function of this covariate . All covariates are centered, the centering (average) value being included as part of the output. This sub-block is only relevant for quantitative traits. It is relevant for the age of onset distribution of a binary trait with variable age of onset, but is not otherwise relevant for a binary trait. In the case, because a logistic age of onset distribution is assumed, the same covariate cannot be specified to modify both the mean and the variance.
- 2. The interaction attribute refers to an interaction with type; the default is to assume no interaction. If there is no interaction, we estimate σ_{AA}^2 , σ_{AB}^2 , σ_{BB}^2 (as many as are specified in the type_var sub-block) and one overall "variance" coefficient for each covariate. If there is interaction, then for this "variance" covariate we estimate an additional two interaction

effects that sum to 0 if two σ^2 parameters are being fitted; and an additional three interaction effects that sum to 0 if three σ^2 parameters are being fitted.

17.3.2.7 The suscept_cov Sub-Block

The following table lists the parameters and attributes that may occur in a suscept_cov sub-block.

- 1. The default is to include no covariates in the analysis. The suscept_cov sub-block indicates which covariates are to modify the logits of susceptibilities or penetrances indicated in the type_suscept sub-block. All covariates are centered, the centering (average) value being included as part of the output. In the case of an age-of-onset distribution, the same covariate cannot be specified as a covariate for both type_mean and the type_suscept.
- 2. The " interaction" attribute refers to an interaction with type; the default is to assume no interaction. If there is no interaction, we estimate β_{AA} , β_{AB} , β_{BB} (as many as are specified in

the type_suscept sub-block) and one overall "susceptibility/penetrance" covariate coefficient for each covariate. If there is interaction, then for this "susceptibility/penetrance" covariate we estimate an additional two interaction effects that sum to 0 if two β parameters are being fitted; and an additional three interaction effects that sum to 0 if three β parameters are being fitted.

 Γ

17.3.2.8 The fpmm Sub-Block

- 1. These parameters cannot be estimated, only specified.
- 2. The following is an example of an fpmm sub-block

fpmm { loci=6 freq=.4 var, val=10.3, fixed=false }

- 3. This sub-block, nested within the fpmm sub-block, may be used to analyze a disease trait with variable age of onset: a bivariate trait in which one trait is binary (affected versus unaffected) and the other is continuous (age of onset) censored for unaffected persons.
- 4. This option is not applicable if the value of loci is set to zero.
- 5. The onset parameter is required for a binary trait with specified (or estimated) age of onset, and is not relevant otherwise.

17.3.2.8.1 The onset Sub-Block

The following table lists the parameters and attributes that may occur in a onset sub-block.

Notes

- 1. The type_dependent values have the following meanings:
	- A Age of onset depends on type.
	- S Susceptibility depends on type.

The option chosen will not cause any dependence on type if A is specified and the type_mean sub-block specifies an option value of one, or if S is specified and the type_suscept sub-block specifies an option value of one.

- 2. The multi_dependent values have the following meanings (must be skipped if loci=0):
	- N There is no polygenic component.
	- A Age of onset has a polygenic component.
	- S Susceptibility has a polygenic component.

If you choose the default option N, neither the age of onset nor the susceptibility depend on the number of polygenic loci.

3. It is permissible for the age_onset and age_exam parameters to specify the same quantitative trait, in which case the value of this trait is assumed to be age of onset for affected persons and age at exam for unaffected persons. This should only be done if the age given for an affected person is the age of onset or unknown, i.e. this disallows the possibility of an affected person having an age at examination when age of onset is unknown. If an affected person has an age of exam but no age of onset, this information cannot be used when age of onset and age of exam are in the same field, so in that situation the result will be different from the result obtained if two separate fields are used.

Example of an onset sub-block nested within an fpmm sub-block:

```
fpmm
{
    loci=6freq=.4
    var, val=10.3, fixed=false
    onset # See onset sub-block below.
    {
        type_dependent=A
        multi_dependent=N
        status=sidease
        age_onset=age
        age_exam=age
    }
}
```
17.3.2.9 The resid Sub-Block

The following table lists the parameters and attributes that may occur in a resid sub-block (see note 1).

- 1. This sub-block is not relevant for the FPMM (finite polygenic mixed model). Residual correlations are relevant for quantitative traits and residual associations are relevant for binary traits.
- 2. The default option value, equal_po_ss, corresponds to the usual genetic mixed model assumption of no marital correlation and equal sib-sib and parent-offspring correlations (only one of the parameters from among mo, fo and ss may be specified)
- 3. With the second value of the option parameter, equal_po, mother-offspring and father-offspring correlations are equal while the father-mother (marital) correlation and sibling-sibling correlation are functionally independent of the parent-offspring correlation and of each other (fm and ss may be specified as well as either mo or fo). With the option value of arb, all four correlations: father-mother, mother-offspring, father-offspring, and sibling-sibling are functionally independent of each other and any combination of these correlations may have their attributes specified.
- 4. The residual value range of $(-1, +1)$ is valid only when modeling quantitative data. In the multivariate logistic model used for non-quantitative data with residuals, the range is a calculated value that changes based on parameter estimates.

17.3.2.10 The transformation Sub-Block

The following table lists the parameters and attributes that may occur in a transformation subblock(see note 1).

- 1. This block is not relevant for a binary trait. The Box and Cox transformation is given in [17.2.2.2](#page-281-0) and the George and Elston transformation is given in 5.2.2.. For the Box-Cox transformation, all values of the trait to which it is applied must be $> -\lambda_2$, and should preferably be $> 1-\lambda_2$.
- 2. The default values $\lambda_1 = \lambda_2 = 1$ result in no transformation when the Box-Cox transformation is applied, and The default values $\lambda_1 = 1$; $\lambda_2 = 0$ result in no transformation when the George-Elston transformation is applied, provided the trait values are all < 1 or all > 1 .

 Γ

17.3.2.11 The geno_freq Sub-Block

Notes

1. The hwe option imposes Hardy-Weinberg equilibrium proportions, nhwe does not.

- 2. If two prob parameters are specified, their sum must be less than 1.
- 3. If true, sufficient information (val attributes of probs_fixed or allele_freq_A, depending on the option chosen) must be specified to fully cover all probabilities. If false and a sufficient number of vals are included to specify all probabilities, they determine initial values of the probabilities. If false and a sufficient number of vals are not included, the program supplies the necessary initial values for the maximization process.

17.3.2.12 The transmission Sub-Block

The following table lists the parameters and attributes that may occur in a transmission sub-block (see note 1).

- 1. This sub-block can only be used if two or three distinct types are specified in either the type_mean or type_suscept sub-block. If this sub-block is missing and a type_mean or type_suscept sub-block is included that specifies two or three types, then all of the option values of this sub-block, with the exception of no_trans, are automatically performed.
- 2. Defining the transmission probability τ_u to be the probability that a person of type *u* transmits A, and *q^A* to be the relative frequency of A, these options correspond to:

- 3. For the 3 " homogeneous" options hwe must be specified in the geno_freq sub-block (or, equivalently, a geno_freq sub-block must not be included).
- 4. This default is appropriate for commingling analysis with the assumption of Hardy-Weinberg equilibrium proportions.
- 5. Note that the hemizygous genotypes A and B are not in the value range. AA and BB are also the values of hemizygous males for X-linked model.
- 6. Does not apply to a tau parameter for which $fixed = true$ or to user-specified initial values. The initial values of the val attribute, if specified, must always lie in the closed interval [0, 1].

17.3.2.13 The ascertainment Sub-Block

The following table lists the parameters and attributes that may occur in a ascertainment subblock.

- 1. This parameter determines whose phenotypes are conditioned on (the "conditioned subset") when calculating a conditional likelihood that allows for ascertainment, as follows:
	- A value of none indicates that unconditional likelihoods are calculated (i.e. no correction for ascertainment – the same as not including this sub-block).
	- A value of psf indicates the members of the pedigree proband sampling frame and is only permissible if a psf_indic parameter is included in the sub-block.
	- A value of founders indicates all founder members of the pedigree. Because founders do not include singletons (see 3.1), this option should not be used if there are any singletons in the data.
	- A value of **founders_plus_psf** indicates all the founder members and the members of the pedigree proband sampling frame, and is only permissible if a psf_indic parameter is included in the sub-block. Because founders do not include singletons (see 3.1), this option should not be used if there are any singletons in the data.
- 2. The cond_val parameter is relevant for quantitative traits only, and is ignored for binary traits, composite traits, and for age of onset models (for which the onset_option parameter in this sub-block should be used). In the case of binary and composite traits, the default value of actual is always used. Also, actual is the value used for all founders not included in the proband sampling frame.

3. The meanings of the values of cond_val are as follows:

- 4. If the value (specified or estimated) of thresh_indic_low is greater than the value of thresh_indic_high, a warning message is printed.
- 5. If cond value is set equal to the value thresh indic, then the value of the threshold indicator variable determines, separately for each individual, which cond_val option to apply. The threshold indicator variable should:
	- be equal to thresh for those individuals for whom **actual** is to be applied.
	- be greater than or equal to thresh for those individuals for whom gte is to be applied.
	- be less than or equal to thresh for those individuals for whom **Ite** is to be applied.
- 6. This parameter is required if a binary trait with variable age of onset is being analyzed (unless random sampling is to be assumed). If set equal to actual, the likelihood is conditioned on the binary trait and actual age of onset for each member of the conditioned subset, if available, otherwise by the age at exam. If the value by onset is specified, the likelihood is conditioned on the binary trait of each member of the conditioned subset and by the actual age of onset, if available, otherwise by the age at exam. However, actual is the value used for all founders not included in the proband sampling frame.
17.3.2.14 The prev_constraints Sub-Block

The following table lists the parameters and attributes that may occur in a prev_constraints sub-block.

17.3.2.14.1 The constraint Sub-Block

The following table lists the parameters and attributes that may occur in a constraint sub-block.

Notes

- 1. Any covariate in this sub-block must also appear in the mean_cov, var_cov or suscept_cov sub-blocks.
- 2. Any covariate (including age) upon which prevalence depends and which is not specified as a covariate parameter, or for which a value is not specified, is set at its mean value.
- 3. It is assumed that, independent of the pedigree data, *R* of *N* persons are affected by the specified values of the covariates. If for a particular specified value of the covariate we have an independent estimate of the prevalence, p, with standard error s.e., then appropriate values of N and R are

$$
N = \frac{p(1-p)}{(s.e.)^2}
$$

and

$$
R = Np.
$$

R and *N* need not be integers.

- 4. The literal string infinity must be entered to indicate an " infinite" age.
- 5. The following example illustrates the constraint syntax:

```
segreg, out = myAnalysis
{
  trait = BMI, type = continuous
  trait = aff, type = age_onset
   .
   .
   .
   prev_constraints
  {
      constraint
      {
         covariate = height
         covariate = weight
         age = infinity
         N = 1000
```
 $R = 100$ } constraint { covariate = smoking covariate = drinking age = infinity } . . . } }

17.3.2.15 The prev_estimate Sub-Block

The following table lists the parameters and attributes that may occur in a prev_estimate subblock.

Notes

- 1. Any covariate in this sub-block must also appear in the mean_cov or suscept_cov sub-blocks. Age of onset (or age at exam) may also be included as a covariate if an onset sub-block is included, and then prevalence is interpreted as the probability of having been affected since birth up to the specified age.
- 2. Any covariate upon which prevalence depends, but is not specified as a covariate parameter, is set at its mean value as indicated in the output. This mean is the average of all the values of the covariate in the sample. Note that if the particular SEGREG run uses other covariates with missing values, causing some individuals to be excluded from the likelihood for that run, the average of the covariate values used for that run may be different from its average in the whole sample.
- 3. The literal string infinity may be entered to indicate an " infinite" age.

17.3.2.16 The output_options Sub-Block

The following table lists the parameters and attributes that may occur in a output_options subblock.

Notes

1. Type probabilities can only be calculated if two or three types are specified in either the type mean sub-block or the type susceptibility sub-block. In either case (1) three probabilities (summing to 1) are output for an individual: the probabilities of being AA, AB or BB conditional on the model and all the pedigree information available, substituting maximum likelihood estimates for all unknown parameters; and (2) for each individual, penetrance functions that can be used as input to LODLINK or MLOD (for autosomal linkage only). Because (2) usually only makes sense if the transmission option homog_mendelian has been chosen, these penetrances will only be produced if that option is among those chosen in the transmission sub-block.

17.4 Program Output

SEGREGproduces several output files that contain results and diagnostic information:

17.4.1 Summary Output File

The SEGREG summary output file stores the table of final estimates of the parameters with model information.

Example:

```
==============================================================================================
SEGREG Analysis for Trait : dbh
==============================================================================================
  # Model Specification
       Model Class A
       Type means <br>Type variances the set one variances in the means in the variance of the set of the set of the set of the set o<br>Set of the set of the
                                                        : one variance
       Genotype frequency Exercise Exercise Exercise 1: Hardy-Weinberg equilibrium
       Residual correlations \cdots : no spouse correlation,
                                                           parent-offspring and sib-sib correlations equal
```


17.4.2 Detailed Output File

The SEGREG Detailed output file stores the variance-covariance matrix as well as what the Summary File has. At the end of the analysis the ln likelihood and -(twice the ln likelihood) are given for the estimated model. Note that these values differ from the true values by a constant that is the same for all analyses performed in the same SEGREG run, but might differ, for the same data, in separate SEGREG runs.

Example:

```
==============================================================================================
SEGREG Analysis for Trait : dbh
==============================================================================================
  # Model Specification
      Model Class A
      Type means<br>
Type variances<br>
Type variances<br>
\frac{1}{2} is the variance
      Type means<br>Type variances<br>Genotype frequency
       Genotype frequency<br>
Residual correlations<br>
\therefore no spouse correlation,
                                                : no spouse correlation,
```


The Detailed Output file also includes a table of p-values for the various transmission models estimated as shown in Figur[e17.1,](#page-331-0) below. *This table is not included as part of the output for X-linked models*.

		Likelihood Ratio Criteria(above diagonal) and Asymptotic P-values(below diagonal) Rows: null hypothesis Columns: model (alternate hypothesis) homo no trans homo mendelian homo general		tau AB free	general
homo no trans			119.8[2]		120.6[3]
homo mendelian	$- - - - -$		0.000[2]	0.812[1]	0.812 $[3]$
homo general	0.000	0.750			0.812[1]
tau AB free	$\frac{1}{2}$	0 184			0.000[2]
general	0.000	0.637	0.368	1.000	
large-sample P-value.		Note: The quoted P-values assume large samples and that the difference in the number of functionally independent parameters estimated in the two models is as indicated in bracket []. Because bounds placed on other parameters in the model used may result in a number that is different, you are cautioned to check this before quoting the corresponding			

Figure 17.1: Transmission Model P-Values

Chapter 18

SIBPAL

This linkage program models trait data from sib pairs as a function of marker allele sharing identityby-descent (IBD). Although this program has been extended to allow for the inclusion of half-sibs in a manner similar to that suggested by Schaid et al. (2000), users are advised to use RELPAL for this situation. Available analyses can use both single- and multi- point IBD information, and models allow for both binary and quantitative traits due to multiple genetic loci, including epistatic interaction and covariate effects. Many options are available for binary traits, including a generalization of the mean test and the proportion test. Like the original SIBPAL, it uses linear regression and hence is extremely fast.

18.1 Limitations

The Haseman-Elston linkage test in this release only supports the univariate analysis of full and half sibling pairs. Full support for multivariate analysis and using other relative pairs is available in the RELPAL program.

Unlike early versions of SIBPAL, this program does not generate IBD sharing estimates itself. That must be done using GENIBD, which outputs an IBD sharing file as input for SIBPAL.

18.2 Theory

18.2.1 Basic Notation

A nuclear family is a set of two individuals who have a mating relationship and their natural children; these children form a full sibship. For this chapter, we define a family to be a connected set of both full sibs and half sibs in a single generation (referred as cluster/string in Schaid et al. (2000)).

Let the number of families in the analysis be *K*.

Let the number of sib pairs (full or half) in the k -th family be n_k , $k = 1, 2, ..., K$.

Let *j* be the index of a sib pair: $j = 1, 2, ..., \sum_{k} n_k = n$, where *n* is the total number of sib pairs.

Conditional on the marker information available, at a particular genomic location let \hat{f}_{1j} be the probability of sharing 1 allele IBD, and \hat{f}_{2j} be the probability of sharing 2 alleles IBD for the *j*-th sib pair. Note that $\hat{f}_{2i} = 0$ in the case of half sib pairs.

Let $\pi = (1+2w_1)/4$ and $\hat{\pi}_j = \hat{f}_{2j} + w_1 \hat{f}_{1j}$ where $0 \le w_1 \le 0.5$ (Whittemore and Tu, 1998), for the *j* -th sib pair. The current default value of *w*¹ is 0.5, corresponding to the mean test for a binary trait.

18.2.2 Test of Mean Allele Sharing

Currently all tests of mean allele sharing are done separately for full and half sib pairs. We first obtain estimates of the means of the $\hat{\pi}_j$ and \hat{f}_{ij} ($i = 0, 1, 2$), which we denote $\bar{\hat{\pi}}$ and $\bar{\hat{f}}_i$, and test the hypothesis that their values agree with expectation under random sampling. These tests are that $E(\bar{\hat{\pi}}) = \pi$ and $E(\bar{\hat{f}}_i) = f_i$, where, when $w_i = 0.5$, $\pi = 0.5$ and $(f_0, f_1, f_2) = (\frac{1}{4}, \frac{1}{2})$ $\frac{1}{2}, \frac{1}{4}$ 4) *for a random sample* of full sib pairs and $\pi = 0.25$ and $(f_0, f_1, f_2) = (\frac{1}{2}, \frac{1}{2})$ $(\frac{1}{2}, 0)$ *for a random sample* of half sib pairs. These means and their variances are estimated by calculating:

$$
\bar{\hat{\pi}} = \frac{1}{n} \sum_{j} \hat{\pi}_{j} \qquad s_{\bar{\hat{\pi}}}^{2} = \frac{1}{n(n-1)} \sum_{j} (\hat{\pi}_{j} - \bar{\hat{\pi}})^{2},
$$

$$
\bar{\hat{f}}_{i} = \frac{1}{n} \sum_{j} \hat{f}_{ij} \qquad s_{\bar{\hat{f}}_{i}}^{2} = \frac{1}{n(n-1)} \sum_{j} (\hat{f}_{ij} - \bar{\hat{f}}_{i})^{2}.
$$

From each mean, a *t* statistic is computed and referred to the *t* distribution with *n* -1 d.f. for a two-sided test. The p-values are

$$
2P\left(t_{n-1}\geq \frac{\left|\bar{\hat{\pi}}-\pi\right|}{s_{\bar{\hat{\pi}}}}\right) \quad \text{and} \quad 2P\left(t_{n-1}\geq \frac{\left|\bar{\hat{f}}_i-f_i\right|}{s_{\bar{\hat{f}}_i}}\right),
$$

and where t_{n-1} is a random variable that is distributed as t with $n-1$ d.f.

18.2.2.1 Test of Mean Allele Sharing for Binary Traits in Selected Pairs

The above tests are also performed separately for pairs with 0, 1, and 2 affected members as tests for linkage. However, all tests are then one-sided and the p-values are

$$
\mathrm{P}\left(t_{n-1}\geq \frac{\delta\left[\bar{\hat{\pi}}-\pi\right]}{s_{\bar{\hat{\pi}}}}\right)\quad\text{and}\quad P\left(t_{n-1}\geq \frac{\delta\left[\bar{\hat{f}}_i-f_i\right]}{s_{\bar{\hat{f}}_i}}\right),
$$

where $\delta = 1$ for concordantly affected pairs (2 affected members) and unaffected pairs (0 affected members), and $\delta = -1$ for discordant pairs (1 affected member). No such tests are performed for full sib pairs if $i = 1$.

18.2.3 Generalized Haseman and Elston Linkage Test

18.2.3.1 Regression model for autosomal markers

The basic model we fit for autosomal markers is of the form:

$$
y = \beta_0 + \sum_m a_m \hat{\pi}_m + \sum_m d_m \hat{f}_{2m} + \sum_c b_c f(z_c) + e
$$

where

- *y* is a dependent variable (see [18.2.3.2\)](#page-335-0),
- β_0 is the intercept,
- $\hat{\pi}_m = \hat{f}_{2m} + w_1 \hat{f}_{1m}$ where the current default value of w_1 is 0.5,
- b_c is a nuisance parameter accounting for the effect of some function f of the c -th covariate term *zc*,
- *e* is the residual error.

In a random sample, when w_1 is 0.5, a_m is the additive genetic variance and d_m is the dominant genetic variance due to the *m* -th autosomal marker. In the case of an autosomal locus, the variances a_m and d_m are the trait locus-specific variances, attenuated by the recombination fraction between the trait and marker loci. Let β be the parameter vector for such a linear model. Then the generalized least squares estimator of $β$ based on the above model is

$$
b = (A'W^{-1}A)^{-1}A'W^{-1}y
$$

and that of the residual variance is

$$
s^2 = \frac{y'W^{-1}(y - Ab)}{n - m}
$$

where *m* is the number of parameters estimated, *n* is the number of sib pairs, *y* is an $n \times 1$ vector of dependent variables with transpose $y' = (y_1, y_2, ..., y_n)$, W is the $n \times n$ weight matrix for *y*, and *A* is an $n \times m$ design matrix - each parameter corresponds to a particular column of *A* . Note that the weight matrix *W* is either a correlation matrix *R* or the residual variance matrix $Σ$ of *y*, depending on the method used to generate the dependent variable *y* (se[e18.2.3.5\)](#page-338-0).

18.2.3.2 Dependent variable y

Let the number of all sibs in the analysis be *N* and the trait values be $x_1, x_2, ..., x_N$.

In the case of a binary trait, $x_i = 1$ for an affected individual and 0 for an unaffected individual, and it is then treated the same way as for any other quantitative trait to obtain the dependent variable.

Let the number of sibs in the *k* -th full sibship be N_k , $k = 1, 2, ..., K'$ (Note: $K' \ge K$).

Let \bar{x} be the mean of the trait x , calculated in one of four possible ways. It can be:

- 1. the sample mean estimated from all the data as $\tilde{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$,
- 2. the population mean or any other value set by the user,
- 3. the sibship specific mean $\tilde{x}_k = \frac{1}{N_k}$ $\frac{1}{N_k} \sum_{i=1}^{N_k}$ $\frac{N_k}{i=1}$ *x*_{*i*}. For an individual who has no full sibs, we set $\bar{x} = \tilde{x}$,
- 4. the best linear unbiased predictor (BLUP) of the sibship mean, $w\tilde{x}_k + (1 w)\tilde{x}$, where

$$
W = \frac{\sigma_b^2}{\sigma_b^2 + \frac{\sigma_r^2}{N_k}}
$$
. Here, σ_b^2 is the variance among full siblings and σ_r^2 is the variance within full siblings. For an individual who has no full sibs, we set $w = 0$.

The default mean is the BLUP of the sibship mean.

In the following, *i* is the index of a sib and *j* is the index of a sib pair. Then the dependent variable for the *j* -th sib pair can be:

$$
y_{j} = \begin{pmatrix} (x_{j1} - \overline{x})(x_{j2} - \overline{x}) & \text{mean-corrected cross-product} \\ -\frac{1}{2} \Big[(x_{j1} - \overline{x}) - (x_{j2} - \overline{x}) \Big]^{2} = -\frac{1}{2} \Big[x_{j1} - x_{j2} \Big]^{2} & -\frac{1}{2} \text{(squared pair trait difference)} & \text{(DIFF)} \\ \frac{1}{2} \Big[(x_{j1} - \overline{x}) + (x_{j2} - \overline{x}) \Big]^{2} & \frac{1}{2} \text{(squared mean-corrected trait sum)} & \text{(SUM)} \\ \text{weighted combination of} & \text{squared pair trait difference} & \text{used for options W2, W3, W4} \end{pmatrix}
$$

In the case of the weighted option W2, the dependent variable for the *j* -th sib pair is computed in the following way:

$$
y_{j} = \frac{\frac{1}{2} \{s_{s}^{2} \left[(x_{j1} - \overline{x}) + (x_{j2} - \overline{x}) \right]^{2} - s_{d}^{2} \left[x_{j1} - x_{j2} \right]^{2} \}}{\left(s_{s}^{2} + s_{d}^{2} \right)},
$$

where

- s_d^2 is the residual variance value from using the dependent variable DIFF,
- s_s^2 is the residual variance value from using the dependent variable SUM.

In the case of the weighted options W3 and W4, the dependent variable vector for a family is computed in the following way:

$$
y = \left(\frac{W_s^{-1}}{s_s^2} + \frac{W_d^{-1}}{s_d^2}\right)^{-1} \left(\frac{W_s^{-1}}{s_s^2} y_s + \frac{W_d^{-1}}{s_d^2} y_d\right),
$$

where

- s_d^2 is the residual variance value from using the dependent variable DIFF,
- s_s^2 is the residual variance value from using the dependent variable SUM,
- *W^d* is the weight matrix from using the dependent variable DIFF,
- *W^s* is the weight matrix from using the dependent variable SUM,
- y_d is the dependent variable vector from using the dependent variable DIFF,
- *y^s* is the dependent variable vector from using the dependent variable SUM.

18.2.3.3 Covariate terms

Let the number of covariates be *C* and the covariate values for *i* -th sib be $z_{i1}, z_{i2}, ..., z_{iC}$; $i =$ 1, 2, ..., *N*.

The *c* -th covariate term for the *j* -th sib pair can be:

.

$$
z_{jc} = \begin{cases} (z_{j1c} + z_{j2c}) & \text{the covariate sum (default)} \\ |z_{j1c} - z_{j2c}| & \text{the covariate absolute difference} \\ (z_{j1c}z_{j2c}) & \text{the covariate product} \end{cases}
$$

where

- z_{j1c} is the raw value for the first sib in the *j* -th sibpair for the *c* -th covariate,
- z_{j2c} is the raw value for the second sib in the *j* -th sibpair for the *c* -th covariate.

Then, the *c* -th covariate term included in the model design matrix A for the *j* -th sib pair is

$$
z_{jc} - \bar{z}_c
$$
 where $\bar{z}_c = \frac{\sum_{j=1}^{n} z_{jc}}{n}$

18.2.3.4 Design matrix A

A is an $n \times m$ design matrix, where m is the number of regression parameters estimated - each parameter corresponds to a particular column of A.

Columns of A^T

- 1. The first one or two columns in the design matrix correspond to intercept(s):
	- (a) In the case when only one type of siblings is allowed, either full or half, the first column is a column of 1s.
	- (b) In the case when both types of siblings are allowed, both full and half, the first column contains 1 for full sib pairs and 0 for half sib pairs, and the second column contains 0 for full sib pairs and 1 for half sib pairs.
- 2. Following this come one or two columns for each marker locus entered in the model:
	- (a) the first of each of these is a column whose elements are $\hat{\pi}_j$, centered on all *n* pairs;
	- (b) the second of each (if present) is a column whose elements are f_2 , centered. For each marker, the user can choose whether or not to include dominance (f_2) in the model; if it is not included, in a random sample a_m is an attenuated locus-specific measure of the total (additive and dominant) genetic variance.
- 3. Following this may come one or more columns each element of which is the product of elements of two (or more) of the previous columns (marker interactions).
- 4. Following this come one or more columns for each covariate entered in the model. Each of these is a column whose elements are the values previously defined in the covariate terms, but centered.
- 5. Additional columns may be entered that are powers of previous covariates or products of previous covariates (covariate interactions). Note that including too many covariate terms may cause A to be singular due to linear dependencies in the data.

Thus, in the case of full sib pairs only or half sib pairs only, A will be of this form (all columns centered except for the first):

In the case when both types of siblings are allowed, full and half, A will be of this form (all columns centered except for the first two):

18.2.3.5 Weight matrix W

18.2.3.5.1 Weight matrix for DIFF, SUM, PROD and W2

The weight matrix for DIFF (W_d), SUM (W_s), PROD (W_p) and W2 (W_2) is the correlation matrix *R* of the corresponding residuals: $W_d = R_d$, $W_s = R_s$, $W_p = R_p$ and $W_2 = R_2$. The correlation matrix *R* is constructed as described in the following section and depends on the type of sib pair used.

18.2.3.5.1.1 Correlation matrix R for full and half sib pairs separately

Let $r_F = (r_{F1}, r_{F0})$ be a vector of residual correlations for full sib pairs, where r_{F0} is the correlation between pairs of full sib pairs sharing 0 sibs in common and r_{F1} is the correlation between pairs of full sib pairs sharing 1 sib in common. *All correlations are between residuals of the regression model* .

Similarly, let $r_H = (r_{H1}, r_{H0})$ be a vector of correlations for related half sib pairs, where r_{H0} is the correlation between pairs of half sib pairs sharing 0 sibs in common and r_{H1} is the correlation between pairs of half sib pairs sharing 1 sib in common.

The vectors r_F and r_H are either estimated from the data for the chosen dependent variable: r_{Fs} and r_{Hs} for SUM, r_{Fd} and r_{Hd} for DIFF, and so on, with the restriction that the correlations are constrained to be greater than 0 to avoid numerical instability, or all set equal to 0 by the user. Additionally, r_{F1} is constrained to b e $\geq r_{F0}$, and $r_{H1} \geq r_{H0}$.

Consider the following family of 5 nuclear families with 7 full sib pairs and 13 half sib pairs.

The correlation matrix **for the chosen dependent variable between pairs of full sib pairs in the** above family is:

The correlation matrix **for the chosen dependent variable between pairs of half sib pairs in the** above family is:

18.2.3.5.1.2 Correlation matrix R for full and half sib pairs combined

To obtain a correlation matrix^{[1](#page-339-0)} for full and half sib pairs combined, the same correlation values as indicated in the two separate matrices above are used for the pairs of full sib pairs and the pairs of half sib pairs, except that the ratio of the residual variances $c = s_H^2/s_F^2$ is used in the diagonal for the half sib pairs, where

- s_F^2 is the residual variance from the regression using full sib pairs only and
- s_H^2 is the residual variance from the regression using half sib pairs only.

¹The structure is referred to as " correlation" matrix even though the diagonal values are not all 1.

The correlations between full sib pairs and related half sib pairs (and related full sib pairs from other sibships within a family) are computed as follows. Analogous to the previous correlation vectors r_F and r_H , let $r = (r_1, r_0)$ be a vector of correlations between full sib pairs and related half sib pairs, where r_0 is the correlation between these sib pairs sharing 0 sibs in common when at least two of the four sibs are related as half-sibs and r_1 is the correlation between these sib pairs sharing 1 sib in common. These correlations are also constrained to be $r_1 \ge r_0 \ge 0$. Note that if none of the four sibs are related as half sibs, the correlation is set to 0.

Thus, the correlation matrix \bf{R} of the chosen dependent variable for combined sib pairs in the above family is:

As for r_F and r_H , all of the non-diagonal elements in **R** can be set to 0 by the user.

18.2.3.5.2 Weight matrix for W3

The weight matrix W_3 for option W3 is the estimated residual variance matrix Σ of the weighted dependent vector y , so that $W_3 = \Sigma = \Big(\frac{1}{s^2}\Big)^2$ s_d^2 $R_d^{-1} + \frac{1}{s_s^2}$ $\frac{1}{s_s^2}R_s^{-1}$ *s* $\big)^{-1}$, where

- s_d^2 is the residual variance value from using the dependent variable DIFF,
- s_s^2 is the residual variance value from using the dependent variable SUM,
- R_d is the correlation matrix from using the dependent variable DIFF,
- *R^s* is the correlation matrix from using the dependent variable SUM.

18.2.3.5.3 Weight matrix for W4

The weight matrix W4 for option W4 is further adjusted for possible non-independence of the squared trait sums and differences.

Let $r_{Fsd} = (r_{Fds2}, r_{Fds1}, r_{Fds0})$ be a vector of correlations between the residuals of the sums and differences for full sib pairs with 2, 1, 0 sibs in common. Similarly, let $r_{Hsd} = (r_{Hds2}, r_{Hds1}, r_{Hds0})$ be a vector of correlations between the residuals of the sums and differences for half sib pairs with 2, 1, 0 sib in common. These correlations are estimated from the data with the restrictions $r_{Fds2} \ge r_{Fds1} \ge r_{Fds0} \ge 0$ and $r_{Hds2} \ge r_{Hds1} \ge r_{Hds0} \ge 0$.

The correlation matrix between y_s and y_d , R_{sd} , is constructed in the in the same way as the correlation matrices above except that the diagonal elements are *rFsd*² or *rHsd*² instead of being all 1. For example, *Rsd* for the full sib pairs and half sib pairs in above family separately will look like the following:

To obtain a correlation matrix for full and half sib pairs combined, the same correlation values as indicated in the above two matrices are again used for the full sib pairs and the half sib pairs, except that the diagonal elements for the half sib pairs are multiplied by the ratio of the residual variance $c = s_H^2/s_F^2$.

Correlations between sums and differences of full sib pairs and related half sib pairs (and related full sib pairs from sibships within a family) are also computed the same as before. Let $r_{sd} = (r_{sd1}, r_{sd0})$ be a vector of correlations between the residuals of the sums and differences for full sib pairs and

related half sib pairs, where r_{sd0} is the correlation between these sib pairs sharing 0 sibs in common when at least two of the four sibs are related as half-sibs and r_{sd1} is the correlation between these sib pairs sharing 1 sib in common. These correlations are also constrained to be $r_{sd1} \ge r_{sd0} \ge 0$. Note that if none of the four sibs are related as half sibs, the correlation is set to 0.

Thus, the correlation matrix R_{sd} for combined sib pairs in the above family is:

Then, the weight matrix W_4 for the option W4 method is the variance matrix Σ , adjusted as follows:

$$
W_4 = \Sigma + \frac{1}{s_s s_d} \Sigma \Big(R_s^{-1} R_{sd} R_d^{-1} + R_d^{-1} R_{sd} R_s^{-1} \Big) \Sigma \,,
$$

where

- s_d^2 is the residual variance value from using the dependent variable DIFF,
- s_s^2 is the residual variance value from using the dependent variable SUM,
- R_d is the correlation matrix from using the dependent variable DIFF,
- *R^s* is the correlation matrix from using the dependent variable SUM.

18.2.3.6 Generalized estimating equations (GEE)

An iterative method using the generalized estimating equations (GEE) of Liang and Ziegler(1986) is used in each model to allow for the non-independence of sibling pairs. Initially, all correlations are set to 0 to obtain the residuals, then the correlations of residuals from the previous iteration are used to update the weight matrix *W*, and new values of the parameter estimates b_i and s_i^2 are generated, $i = 1, 2, ..., m$.

Let \tilde{r}_0 , \tilde{r}_1 be residual correlations from the previous iteration and be these calculated in the current iteration. The iteration is stopped when the value $\delta =$ *r*˜0−*r*⁰ \tilde{r}_0 $|+|$ *r*˜0−*r*⁰ \tilde{r}_0 $\vert \leq 0.2$ or the maximum number of iterations is achieved.

18.2.3.7 Significance tests

To assess significance, we use the *t* test statistics $\frac{b_i}{v_i}$, for $i = 2, 3, ..., m$, where b_i is the *i*-th element of the parameter estimates $b = (A^T W^{-1} A)^{-1} A^T W^{-1} y$ and v_i^2 is the variance estimate, which is the product of the *i* -th diagonal element of $(A^T W^{-1} A)^{-1}$ and $\frac{y^T W^{-1} (y - Ab)}{n-m}$ $\frac{(y-\Delta U)}{n-m}$.

For each test statistic we calculate a p-value which is either

$$
p_t = \mathbf{P}\left(t_{n-m} \ge \frac{b_i}{v_i}\right) \quad \text{(one-sided test)}
$$

or

$$
p_t = 2\mathbf{P}\left(t_{n-m} \geq \frac{|b_i|}{v_i}\right) \quad \text{(two-sided test)},
$$

where *tn*−*^m* is a random variable that is distributed as *t* with *n-m* d.f.

Estimates b_i corresponding to a column of $\hat{\pi}$ s or \hat{f}_2 s and other columns of marker terms (i.e., products of $\hat{\pi}$ s or \hat{f}_2 s) use one-sided tests. A two-sided test is used for all remaining columns that contain any covariate terms.

Furthermore, the above tests can be performed using variances estimated using an estimator that is robust to misspecification of the model and the correlation matrices. When this option is specified, the covariance matrix of the parameter estimates is computed using the *sandwich* variance estimator

$$
(A'W^{-1}A)^{-1}\left[A'W^{-1}(y-Ab)\right]\left[A'W^{-1}(y-Ab)\right](A'W^{-1}A)^{-1}
$$

and use the same *t* test statistics $\frac{b_i}{v_i}$ as above, except v_i^2 is now the *i*-th diagonal element of the *sandwich* variance matrix estimate. These variance estimates can be extremely conservative and caution should be exercised when using this option. These need only be used when the data contain full sibs.

18.2.3.8 Empirical estimates of significance (full sibs only)

We can also estimate an empirical mid p-value of the test statistic using a Monte Carlo permutation procedure with N replicate permutations. For each replicate, we permute the allele sharing among the pairs (both within sibships and across sibships of the same size), recalculate the test statistic, and determine the proportion of the replicates that have have p-values less than, plus half the proportion that have p-values equal to the p-value calculated from the original observations. We choose N, the number of replicates, such that the estimated empirical mid p-value, \hat{p} , is within a proportion *w* (the width parameter) of its true p-value, p , with predetermined confidence probability (the confidence parameter). That is, we want the standard deviation $s_{\hat{p}}$ of \hat{p} to be proportional to \hat{p} . This permutation process can be viewed as a set of N independent Bernoulli trials each with success probability *p* . The sample variance, $s_{\hat{p}}^2$, of \hat{p} is $s_{\hat{p}}^2 = \frac{\hat{p}(1-\hat{p})}{N}$ $\frac{p(p)}{N}$. So we choose N such that $Pr(|\hat{p} - p| \leq w\hat{p}) = \gamma$. Using a normal approximation for the distribution of \hat{p} , we obtain

$$
N = \left(\frac{1-\hat{p}}{w^2\hat{p}}\left[\Phi^{-1}\left(\frac{\gamma+1}{2}\right)\right]^2\right),\,
$$

,

where Φ is the standard normal cumulative distribution function. We estimate N by substituting for \hat{p} the p-value obtained on assuming the test statistic follows a *t* distribution, and use this number of replicates to obtain an empirical p-value within the pre-specified proportion *w* of its true value with confidence coefficient γ . For example, if we wish to estimate an empirical p-value that is within 20% of its true value with 95% confidence, then N should be approximately $\frac{100(1-\hat{p})}{\hat{p}}$. The number of replicates, N, can be limited to avoid excessive computing time.

18.2.3.9 Regression model for X-linked markers

Note that this is implemented for full-sibs only.

In the case of X-linkage, there are three types of sib pairs, and hence three different πs (Wiener *et al* . 2003):

$$
\hat{\pi}_{\text{BB}} = \hat{f}_0 \text{ for brother - brother pairs}
$$
\n
$$
\hat{\pi}_{\text{BS}} = \hat{f}_0 \text{ for brother - sister pairs}
$$
\n
$$
\hat{\pi}_{\text{SS}} = \hat{f}_1 \text{ for sister - sister pairs}
$$
\n
$$
\text{all are in the model.}
$$

At the X-location, the three coefficients correspond to three different variance components, just as in the autosomal case the coefficient of $\hat{\pi}$ is σ_g^2 .

Let the three coefficients be β_{BB} , β_{BS} , and β_{SS} . Each is tested by a t-test (one-sided), analogous to the autosomal case. If all three are > 0 , we do a 3-d.f. F-test. If only two are > 0 , we do a 2-d.f. F-test.

For the 3-d.f. F-test, we do two regressions, one with the 3 $\hat{\pi}$ s in the model and one without. Call the corresponding residual sums of squares SSE_C (complete model) and SSE_R (reduced model), respectively. Then the F statistic will be

$$
\frac{SSE_R - SSE_C}{3MSE_C}
$$
 with 3 and *N-p* d.f.

where

- *N* is the effective number of independent sib pairs (= total number of sibs number of sibships) and
- *p* is the total number of parameters for full model (= number of columns in the design matrix that includes the 3 $\hat{\pi}$ s).

For the 2-d.f. F-test, we perform an analogous test for the two coefficients that are positive, but treating the parameter that has a negative coefficient as a nuisance parameter, i.e. included in both models to obtain *SSE^C* and *SSER*.

Permutations to get empirical p-values are done by permuting within each of the 3 kinds of sib pairs, as well as within and across sibships as done for the autosomal cases, and calculating the above statistics for each permutation replicate.

18.3 Program Input

18.3.1 Running sibpal

A typical run of the SIBPAL program may use flags to identify the file types like the following:

>sibpal -p data.par -d data.ped -i ch1.ibd

or, rely on a set file order like the following:

>sibpal data.par data.ped ch1.ibd

where data.par is the name of the parameter file, data.ped is the name of the pedigree data file, and ch1.ibd is the name of the IBD sharing file.

18.3.2 The sibpal Block

A sibpal block in the parameter file sets the options on how to perform an analysis using SIBPAL. The following table shows the syntax for a sibpal parameter which starts the sibpal block.

The following table lists the parameters and attributes that may occur in a sibpal block.

Notes

1. If a trait_regression statement does not have either the single or multiple attribute, then the trait_regression_default statement will determine whether the given marker or interval estimates will be regressed one at a time (single) or all at once (multiple).

Each trait_regression statement performs a test of linkage of a trait to one or more markers. The analysis may consist of several regression tests each using a single marker, if either the single attribute is included or the value of the trait_regression_default parameter is set to single. Similarly, a single multiple-regression test is performed if either the multiple attribute is included or the value of the trait_regression_default parameter is set to multiple. The traits, covariates, markers and other options to be used may be listed in a sub-block of the trait_regression statement*.* All options changed in a sub-block are local to the analysis being performed, and do not affect further analyses. If no sub-blocks are listed, then analysis will be performed using all traits and all markers. All parameters that may be included in the sub-block are optional and all values are caseinsensitive.

- 2. Single regression is performed by default.
- 3. This performs the regression analysis without any marker in the regression model.

18.3.2.1 The mean_test Sub-Block

The following table lists the parameters and attributes that may occur in a mean_test sub-block.

Notes

1. The value of a marker parameter should be set to the name of a marker for which IBD sharing information was generated and stored in the IBD sharing file. If no valid marker parameters are listed, then all markers are used. The following are all valid mean_test statements:

```
mean_test # Test each marker
mean_test # Equivalent to the previous statement.
{
}
mean_test
{
 marker=M1
 marker="region 1 MRK"
  marker=M3
}
```
2. The subset parameter specifies a trait to be used as a binary variable to limit the individuals that may be used in an analysis; individuals for whom this indicator is zero are assumed to have missing trait values. It may be included more than once, in which case the only individuals included in the analysis are those for which all the indicated binary traits are coded 1. The trait being analyzed for linkage should not be used as a subset variable.

- 3. If the wide_out parameter is set to true, then additional columns are added to the output from Trait Regression analyses, including a column of t-values corresponding to each parameter estimate.
- 4. The value of w1 cannot be specified (i.e, it keeps its default value 0.5) if half-sibs are being analyzed. Specifying this value to be 0 when the sample analyzed consists of only full sibs leads to the "proportion test" when the mean test is performed.

18.3.2.2 The trait_regression Sub-Block

The following table lists the parameters and attributes that may occur in a trait_regression sub-block.

Notes

- 1. The value of a trait parameter should be set to the name of a trait or covariate field read from the data file or created by means of a function block. If no valid trait parameters are listed, then all trait fields read in are used. If more than one trait is specified then multiple univariate regressions are performed using each trait with all markers and covariates listed. The *population* mean of the trait may be used in computing the mean-corrected trait values. This is specified by including an attribute, mean, with value set to the desired trait mean. Other options, sample mean of individuals used in the regression, using sibship-specific mean or using the best linear unbiased predictor (BLUP) mean, can be specified by setting mean = sample, mean = sibship or mean = blup. If not specified, the best linear unbiased predictor (BLUP) mean is used by default.
- 2. The value of a marker parameter should be set to the name of a marker for which IBD sharing information was generated by GENIBD and stored in the IBD sharing file. If no valid marker parameters are listed then all markers in the IBD sharing file are used.
- 3. If a marker parameter has the dom or dominance attribute, then the additive and dominance variances due to that marker will be tested separately (i.e. there will be regression on both $\hat{\pi}$ and \hat{f}_2); and a marker parameter without this attribute will test total genetic variance due to that marker (i.e. there will be regression on $\hat{\pi}$ only).
- 4. The value of a covariate parameter should be set to the name of a trait or covariate field read from the data file or created by means of a function block. If no valid covariate parameters are listed, then by default no covariates are included. The covariate, with its attribute, will be raised to the specified power *before* mean correction is applied.
- 5. The interaction parameter should contain a sub-block of marker and covariate parameters that specify a multiplicative interaction term in the regression model. Note that interaction terms are allowed only when both corresponding main effects are included in the model, and marker by marker interaction is only allowed in multiple_marker trait regression. The following interaction sub-block specifies a gene-environment interaction term between the dominance component of D1S344 and the squared BMI difference:

```
interaction
{
 marker = D1S344, domcovariate = BMI, diff, power = 2}
```
6. The values for the regression_method are explained as follows:

*^a*This method should be more powerful asymptotically (see Shete, et al., 2003)

*^b*This method should be the most powerful asymptotically (see Shete, et al., 2003)

A general recommendation might be to try using the the last regression method in this list (W4) first, and then work upwards until no signs of numerical instability are seen. The default is set at **prod** in case the user does not know how to recognize such instability, which is very unlikely to occur for prod. Unlike all the other options, the results from the simplest option, diff, are not affected by the sample mean.

- 7. The trait specified by a subset parameter should be a binary trait coded as 0 for individuals to be excluded from, and 1 for individuals to be included in, the analysis. The subset parameter may be included more than once, in which case the only individuals included in the analysis are those for which all the indicated binary traits are coded 1.
- 8. If the wide_out parameter is set to **true**, then additional columns are added to the output from Trait Regression analyses, including a column of t-values corresponding to each parameter estimate.
- 9. If either the zero_marker or multiple_marker attribute is specified for the trait_regression parameter, then no value is required to specify the location. If the single_marker attribute is specified, then a character string representing the name of a marker or location listed in the IBD sharing file must be used to specify the location to print. If no value is specified for single marker regression, then the first n rows of the design matrix for all locations will be printed.
- 10. If either the zero_marker or multiple_marker attribute is specified for the trait_regression parameter, then no value is required to specify the location. If the single_marker attribute is specified, then a character string representing the name of a marker or location listed in the IBD sharing file must be used to specify which location to print.
- 11. This option is not relevant if the zero_marker attribute is specified for the trait_regression parameter. If the multiple_marker attribute is specified, then no value is required to specify the location. If the single_marker attribute is specified, then a character string representing the name of a marker or location listed in the IBD sharing file must be used to specify which location to print. The QLS output lists, for each sib-pair:
- the pedigree ID
- a number indicating the sibship within the pedigree
- the IDs of the two sibs
- a number indicating the sibship within the pedigree
- (type of affected status of sib-pair, if the trait is binary)
- the dependent trait in the HE regression, standardized
- the quamtitative linkage score (QLS) for each sib-pair, averaged over the sib-pairs in the sibships.

The linkage score in Wang and Elston (2006) is then obtained by adding together all the scores in a pedigree. However, this will be proportional to the score as defined by Wang and Elston (2006), rather than the score they defined. Thus it can be used for the same purpose, but the actual value will not be the same.

12. The value of w1 cannot be specified (i.e, it keeps its default value 0.5) if half-sibs are being analyzed, i.e., if the value of use_pairs parameter is set to either half or both.

18.4 Program Output

SIBPAL produces several output files that contain results and diagnostic information:

18.4.1 Mean Analysis Output File

One Mean Analysis output file, named "means.out", is generated per run of SIBPAL. It includes the results of all non-trait specific mean analyses: average allele sharing, as well as 0, 1 and 2 alleles IBD are output in a table with standard errors and p-values for each estimate.

Example:

```
======================================================================
 Test of Mean Allele Sharing IBD for Full Sib Pairs
======================================================================
 Estimates:
   pi - Average proportion of alleles shared IBD.
   fi - Estimated proportion of sib pairs sharing i alleles IBD.
 w1: 0.50
======================================================================
Marker Pairs Estimate Std Error P-value
----------------------------------------------------------------------
D5G1 28 pi 0.45535714 0.04820449 0.36259122038
                    f0 0.29464286 0.05310089 0.40789006347
                     f1 0.50000000 0.02571722 -------------
                     f2 0.20535714 0.04645782 0.34511298675
----------------------------------------------------------------------
D5G2 28 pi 0.43750000 0.05696380 0.28224850647
                      f0 0.30357143 0.07426574 0.47689606098
                      f1 0.51785714 0.07919128 -------------
                      f2 0.17857143 0.06410910 0.27502563114
----------------------------------------------------------------------
.
.
.
```
18.4.2 Trait Regression Analysis Summary Output File

One Trait Regression analysis output file, named "traits.out", is generated per run of SIBPAL. It contains the results of all Trait Regression linkage tests. Each coefficient estimated is printed in a table with its standard error and p-value.

Example:

```
==============================================================================
Haseman-Elston Regression Analysis of Full Sibs
 - single_marker regression
       ==============================================================================
   Binary trait : affection, affected = 'A', unaffected = 'U'Dependent variate : Mean-corrected squared trait sum
   Other options used :
     Identity weights = no
     Robust variance = no
     Use sibship mean = no
     Use BLUP mean = no
   Legend :
     Note - kurtosis = coefficient of kurtosis - 3
     * - significance .05 level;
     ** - significance .01 level;
     *** - significance .001 level;
     # - Negative intercept set to 0;
==============================================================================
Test Independent and the set of the Nominal Community of the Nominal Assembly of the Nominal Assembly of the No
No. variable Pairs Parameter Estimate Std Error P-value
----- ----------------- ----- ----------- ----------- --------- ---------
1 D5G1 28 (A+D)GenVar 0.0108 0.0619 0.4311918
2 D5G2 28 (A+D)GenVar 0.0109 0.0523 0.4186353
3 D5G3 28 (A+D)GenVar -0.0091 0.0561 0.5638613
4 D5G4 28 (A+D)GenVar -0.0139 0.0638 0.5852159
5 D5G5 28 (A+D)GenVar -0.0397 0.0474 0.7952521
6 D5G6 28 (A+D)GenVar -0.0064 0.0536 0.5474505
7 D5G7 28 (A+D)GenVar -0.0568 0.0589 0.8284099
1 DSG1<br>8 D5G8 28 (A+D)GenVar 0.0000 0.0643 0.5000000<br>9 D5G9 28 (A+D)GenVar 0.0436 # 0.0491 0.1909162
9 D5G9 28 (A+D)GenVar 0.0436 # 0.0491 0.1909162
10 D5G10 28 (A+D)GenVar -0.0000 0.0580 0.5000000
11 D5G11 28 (A+D)GenVar -0.0036 0.0564 0.5251796
12 D5G12 28 (A+D)GenVar -0.0074 0.0556 0.5521124
.
.
.
24 D5G24 28 (A+D)GenVar -0.0120 0.0522 0.5899965
25 D5G25 28 (A+D)GenVar -0.0803 0.0472 0.9497509
==============================================================================
```
18.4.3 Trait Regression Analysis Detailed Output File

One Trait Regression analysis output file, named "traits.det", is generated per run of SIBPAL. It contains the detailed results of all Trait Regression linkage tests.

Example:

```
=====================================================================================
Test 1
=====================================================================================
Model
-----
affection ~ Intercept + D5G1(A+D) + e
------
Sample
------
Number of all sibs = 27
Number of full sib pairs = 28
-----
Trait
-----
\begin{array}{cccc}\n\text{Sample mean} & = & 0.0370 \\
\text{Sample variance} & = & 0.0370\n\end{array}Sample variance = 0.0370Sample skewness = 4.9029
Sample kurtosis = 22.0385
Pairwise full sib correlation = -0.0182
Intra sibship correlation = 0.0610
-----------------
Dependent variate
-----------------
Correlation between pairs with no sibs in common = 0.0000
Correlation between pairs with one sib in common = 0.0000
Correlation between squared difference
               and squared mean corrected sum = -1.0000
----------
Regression
----------
                  Estimate Std Error P-value
 ----------------- --------- --------- ---------
Intercept 0.0130
D5G1 0.0108 0.0619 0.4311918
Total variance 0.0062
Residual variance 0.0067
Residual skewness 4.9942
Residual kurtosis 22.9761
=====================================================================================
Test 2
=====================================================================================
.
.
.
```
Chapter 19

TDTEX

The transmission-disequilibrium test (TDT) introduced by Spielman et al. (1993) is a method for detecting linkage between a marker locus and a disease susceptibility locus when linkage disequilibrium or any other type of allelic association is present. The basic TDT test for binary traits has been generalized by Bickeböller and Clerget-Darpoux (1995), Rice et al. (1995), Curtis and Sham (1995), Olson et al. (1997). TDTEX is a computer program based on this work, and implements a very general system for detecting linkage in the presence of linkage disequilibrium between a marker locus and a disease locus affecting a binary trait. It is a valid test for association in the presence of linkage only if there is only one offspring in each family.

19.1 Limitations

The TDTEX program makes the following assumptions:

- 1. Each marker transmits alleles in a Mendialian fashion.
- 2. Only autosomal loci are considered.
- 3. Only binary traits are considered.

This program is limited by the program execution time of the computer on which it runs. As the transmission table size and number of marker alleles increase, processing time becomes slower. The major computational limitation is the exact permutation algorithm. This becomes prohibitively slow for transmission tables with more than around 300 observations, or with more than about 8 alleles. In such cases, the asymptotic or Monte Carlo test statistics are recommended instead.

19.2 Theory

TDTEX consists of four main components:

1. A scoring algorithm to identify which alleles or genotypes are transmitted to affected offspring.

- 2. Production of transmission tables (i.e., contingency tables) to summarize the number of transmitted vs. non-transmitted alleles or genotypes.
- 3. A pedigree sampler to identify and collect informative transmissions from pedigree data. The sampler collects transmission information in transmission tables, conditional on the types of relatives to be sampled (individual affected offspring or affected sibling pairs), the availability of marker data, and optionally on parental traits such as sex or affection status.
- 4. A suite of statistical tests to evaluate significance of the computed transmission tables under the null hypothesis of complete symmetry or marginal homogeneity. These tests include the standard asymptotic TDT tests which rely on large sample theory for validity. Exact tests that do not rely on asymptotic approximations are also provided at the expense of greater computational requirements.

19.2.1 Allele and Genotype Transmissions

Consider a sample of affected individuals and their parents typed for a genetic marker. The basis of the transmission-disequilibrium test is a case/control study, matching alleles found in an affected individual with internal family-based control alleles. The " case" alleles are those that were transmitted to an affected individual, and " control" alleles are the alleles not transmitted from the parents of the individual. By scoring these transmitted and non-transmitted alleles from pedigree data, it is possible to estimate the distribution of these transmissions. If the marker and trait loci are unlinked or are unassociated (in equilibrium), then the distribution of parental alleles transmitted to affected offspring will not differ in expectation from that of alleles that were not transmitted to the affected offspring. Otherwise, if *both linkage and disequilibrium* (or, more generally, linkage and allelic association, whatever the cause of that association) are present between marker and trait loci, then the distribution of alleles transmitted to the affected offspring will differ from that of the nontransmitted alleles. This approach has the advantage of being robust to the presence of population stratification, a situation caused by admixture of populations with distinct marker allele and disease frequencies. For more details see Spielman et al. (1993).

We define an *allele transmission* from a single parent to a child to be an ordered pair of alleles, where the first allele is transmitted from the parent to the child and the second allele is the other parental allele, i.e., the one that is not transmitted to the child. In other words, an allele transmission is the ordered pair (A_1, A_2) where A_1 is the transmitted allele, and A_2 is the non-transmitted allele.

It is possible to combine the information from the allele transmissions from each of the two parents to a child. Since two allele transmissions involve two transmitted alleles (and two non-transmitted alleles), we can group the transmitted (and non-transmitted) alleles together to form a genotype. Thus a *genotype transmission* is defined as an ordered pair of genotypes, where the first genotype is formed by the two alleles transmitted from the parents to the child, i.e., the genotype of the child. Similarly, the second genotype includes the two alleles not-transmitted from the parents to the child. Consider a pair of allele transmissions from the two parents, (A_1, A_2) and (A_3, A_4) . We denote a genotype transmission from these parents as $(A_1/A_3, A_2/A_4)$, where A_1/A_3 is the transmitted genotype and A_2/A_4 is the non-transmitted genotype (see Figur[e19.1\)](#page-363-0).

Figure 19.1: Allele and Genotype Transmission Examples

19.2.2 Scoring affected offspring

Scoring affected offspring requires computing the allele or genotype transmissions from the parents of an affected individual. However, not all such transmissions are informative and, in the presence of missing parental data, some transmissions cannot be used due to potential bias introduced by population stratification (Curtis and Sham, 1995). Table 19.1 presents the transmissions that are scored and used by TDTEX for affected offspring. The basic distinct patterns of allele configurations for parents and children are shown, together with the resulting allele and genotype transmissions. All possible configurations can be obtained from these by relabeling alleles, permuting the two parents, or permuting the alleles within individuals.

Empty cells in the table represent uninformative or unusable transmissions. Notice that some information on allele transmission can be obtained from affected individuals with only one typed parent.

In this situation, it is known that either an A allele is transmitted and a B allele is not transmitted, or vice versa (see Sham et. al. 2000). In the case of sex-specific counting, however, this type of transmission is not considered informative (since it is not known from which parent an individual received a particular allele).

19.2.3 Scoring affected sibling pairs

In some situations it is advantageous to test for linkage disequilibrium in data sets consisting of pairs of affected offspring and their parents (Spielman et al., 1993; Cleves et al., 1997). This

Parent		Parent		Child	Parent 1	$\overline{2}$ Parent	Genotype
1		2			transmis-	transmis-	transmis-
					sion	sion	sion
A/A	X	A/A	\rightarrow	A/A			
A/A	\mathbf{x}	A/B	\rightarrow	A/A	A, A	A,B	A/A , A/B
A/A	X	A/B	\rightarrow	A/B	A, A	B, A	A/B , A/A
A/A	X	B/B	\rightarrow	A/B	A, A	B,B	A/B , A/B
A/A	\mathbf{x}	B/C	\rightarrow	A/B	A, A	B, C	A/B, A/C
A/B	X	A/B	\rightarrow	A/A	A,B	A,B	A/A , B/B
A/B	X	A/B	\rightarrow	A/B	$*A, B$	$*A, B$	$*A/B, A/B$
A/B	X	A/C	\rightarrow	A/A	A,B	A, C	A/A , B/C
A/B	X	A/C	\rightarrow	A/B	B, A	A, C	A/B, A/C
A/B	X	A/C	\rightarrow	B/C	B, A	C, A	B/C , A/A
A/B	X	B/C	\rightarrow	A/B	A,B	B, C	A/B , B/C
A/B	X	C/C	\rightarrow	A/C	A,B	C, C	A/C , B/C
$\frac{2}{2}$	X	A/A	\rightarrow	A/A			
2/2	X	A/A	\rightarrow	A/B		A, A	
$\frac{2}{3}$	X	A/B	\rightarrow	A/A			
2/2	\mathbf{x}	A/B	\rightarrow	A/B			
2/2	X	A/B	\rightarrow	A/C		A,B	
$\frac{2}{3}$	\mathbf{x}	$\frac{2}{2}$	\rightarrow	A/A			
2/2	X	$\frac{2}{2}$	\rightarrow	A/B			

Table 19.1: Transmissions scored for all possible distinct configurations of parents and offspring.

Figure 19.2: Allele and Genotype Transmission for Sibling Pairs

variant of the TDT scores only the same allele transmissions to both affected offspring. This is a narrower sampling scheme than the standard affected offspring version, because transmissions from heterozygous parents that transmit a different allele to each offspring are ignored. In some situations, sampling affected sib pairs rather than affected individuals greatly improves the power of the TDT (see Figur[e19.2\)](#page-365-0).

Table 19.2 presents the transmissions that are scored and used by TDTEX for affected sibling pairs. The basic possible allele configurations for parents and children are shown, together with the resulting allele and genotype transmissions. Empty cells in the table represent uninformative or unusable transmissions. Notice that some information on allele transmission can be obtained from affected pairs with only one typed parent.

* - parental origin is unknown, we cannot know whether the same allele is transmitted from a given parent to both children.

19.2.4 Transmission Tables

To test for differences between the distribution of transmitted alleles and genotypes and non-transmitted alleles and genotypes, TDTEX tabulates all the pairs of transmissions and non-transmissions into contingency tables, henceforth called "transmission tables".

Let M_1 .. M_K represent the K alleles or genotypes at a given marker locus. Transmission tables are defined to be K x K tables of counts, where the rows represent transmitted alleles or genotypes, and columns are the non-transmitted alleles or genotypes (Tabl[e19.3\)](#page-367-0). The entries n_i are the number of times M_i was transmitted and M_i was not transmitted to an affected individual/pair.

The diagonal elements of the table, (when scoring allele transmissions, those from homozygous parents) contain no information and are ignored in the analysis.

Non-transmitted										
Transmitted	${\bf M}_1$	$M_2 \dots$		M_K	Total					
\mathbf{M}_1	n_{12}	n_{12}	\cdots	n_{1K}	$n_{1\bullet}$					
M_2	n_{21}	n_{22}	\ldots	n_{2K}	$n_{2\bullet}$					
M_K	n_{K1}	n_{K2}	\cdots	n_{KK}	$n_{K\bullet}$					
Total	n_{\bullet} 1	n_{\bullet}	\mathbf{r} , \mathbf{r} , \mathbf{r}	$n_{\bullet k}$	$n_{\bullet\bullet}$					

Table 19.3: The structure of a transmission table

19.2.5 Pedigree sampler

The pedigree sampler is the component of TDTEX that controls the construction of transmission tables. It traverses the pedigree data, identifies potentially informative individuals and pairs based on trait and marker data, scores them, and tabulates the results into a transmission table. For each nuclear family, the sampler first attempts to find any *informative* affected sibling pairs, up to a user-specified maximum number. This maximum can be set to zero to disable the sampling of affected sibling pairs, or to an unlimited value to select as many as possible. The sampler will only allow each child to participate in at most one transmission, so there is no problem with overlapping affected sibling pairs. The remaining offspring not already used in a sibling pair are then scored, up to a separate user-specified maximum number. This maximum can also be set to zero to disable the sampling of affected sibling pairs, or to an unlimited value to select as many as possible.

The traditional TDT test corresponds to setting the maximum number of affected children per nuclear family to 1 and the maximum number of affected sibling pairs to none. The sampler will then score the first informative allele or genotype transmission to an affected offspring, and then move on to score the next nuclear family. This will result in a valid test of allelic association in the presence of linkage.

Some other implementations of the TDT test work by setting the maximum number of affected children per nuclear family to unlimited, and the number of affected sibling pairs to none. This allows the sampler to score all informative affected offspring in each nuclear family. Similarly, basic TDT tests utilizing only sibling pairs are possible by setting the maximum number of affected

offspring to none, and the maximum number of affected sibling pairs to 1 or unlimited. This will result in valid tests of linkage in the presence of allelic association.

An interesting option exists to enable the sampling of both affected sibling offspring *and* affected sibling pairs. This very general variation gives preference to informative affected sibling pairs over affected offspring. Overall, this configuration provides a way to take advantage of more information from datasets that include a mixture of family types, not all of which have two affected offspring. Equal weight is given to all transmissions, so power may not be optimal in spite of the larger sample size.

19.2.6 Testing significance of transmission tables

Two null hypotheses have been proposed to test transmission tables for deviations from the expected pattern of allele and genotype transmissions. The first hypothesis is that of complete symmetry between the transmitted and non-transmitted alleles. This states that the expected number of any transmission type is equal to the expected number of transmissions of the opposite pattern, i.e., $E(n_{ij}) = E(n_{ij})$. The second hypothesis is the hypothesis of marginal homogeneity: in this case, the number of alleles or genotypes transmitted is compared to the number not transmitted, i.e., $E(n_{i_{\bullet}})$ = $E(n_{\bullet i})$. Which null hypothesis is optimal depends on the sample size, number and distribution of alleles, and the structure of the disequilibrium present in the sample. TDTEX provides tests based on both hypotheses for maximum flexibility.

TDTEX also includes both exact and asymptotic tests. Exact tests, as the name suggests, provide exact significance levels at the expense of being computationally intensive. Asymptotic tests are based on distributional theory and approximations that are only precise for very large sample sizes. They tend to be very quick to compute, but there are situations when asymptotic tests are significantly less powerful than exact versions. Typically, this occurs when sample sizes are small, transmission tables are sparse, and cells have less than 5 observations.

Statistics based on both the hypotheses of complete symmetry and marginal homogeneity may be applied to tables of allele transmissions as well as genotype transmissions. Genotype transmission tables may be preferred because the transmission patterns of the two parents, which include transmission from the homozygous parents, are not independent in the multiallelic case, except when linkage is complete (Bickeböller and Clerget-Darpoux, 1995). However, because of the larger size and increased sparseness of genotype transmission tables for markers with multiple alleles, the marginal homogeneity test is less prone than the complete symmetry test to problems arising from table sparseness.

19.2.6.1 Asymptotic Tests

Under the hypothesis of complete symmetry, the McNemar test statistic

$$
T_{mc} = \sum_{i < j} \frac{(n_{ij} - n_{ji})^2}{n_{ij} + n_{ji}} \sim \chi^2_{K(K-1)/2}
$$

has an asymptotically χ^2 distribution with K(K - 1)/2 degrees of freedom (Bickeböller and Clerget-Darpoux, 1995). In practice, the number of degrees of freedom equals the number of types of parental heterozygotes in the sample. A continuity corrected version of the McNemar test statistic

$$
T_{mcc} = \sum_{i < j} \frac{(|n_{ij} - n_{ji}| - 1)^2}{n_{ij} + n_{ji}} \sim \chi^2_{K(K-1)/2}
$$

is also provided, since it tends to be more robust to small sample sizes.

Under the hypothesis of marginal homogeneity, the test statistic

$$
T_{mh} = \frac{K-1}{K} \sum_{i} \frac{(n_{i.} - n_{.i})^2}{n_{i.} + n_{.i} - 2n_{ii}} \sim \chi^2_{K-1}
$$

has an asymptotically χ^2 distribution with K-1 degrees of freedom often, provided the table margins are independent of each other (Spielman and Ewens, 1996).

19.2.6.2 Exact tests

The exact test of complete symmetry or marginal homogeneity is generally a more powerful test than the asymptotic tests in the presence of table sparseness and/or a small sample size. To obtain the null permutation distribution for the exact test, we write the distribution of the n_i , conditional on the sums of complementary off-diagonal cells, as the product of K(K-1)/2 binomial random variables with equal probability of transmission vs. non-transmission:

$$
Pr(n) = \prod_{i < j} \binom{n_{ij} + n_{ji}}{n_{ij}} \left(\frac{1}{2}\right)^{n_{ij} + n_{ji}}
$$

An exact significance level is determined by calculating the probability of finding a permutation of the observed data, conditional on the sums of complementary off-diagonal cells, that is as extreme as, or more extreme than, the observed transmission table. Let $N = \{n' : n'_{ij} + n'_{ji} = n_{ij} + n_{ji}\}$ be the set of all permutations of the observed data, conditional on the sums of complementary off-diagonal cells. Let $N' = \{n' : Pr(n') \leq Pr(n), n' \in N\}$, be the set of all permutations with probability less than or equal to that of the observed data. Then the significance level, or p-value, is $P_{cs} = \sum Pr(n').$ $n \overline{\in} N$ ⁰

Since enumerating all possible permutations of the observed transmission table is infeasible for larger tables, the exact permutation algorithm relies upon methods of ordering permutations of the observed table, and by avoiding the evaluation of many equivalent tables. The algorithm uses the fact that the probability after permuting a pair of symmetric odd-diagonal cells in a transmission table does not involve the remaining cells. The null probability distribution is also independent of the direction of asymmetry. For example, a configuration in which $n_{12} = 4$ and $n_{21} = 0$ has the same probability as that of $n_{12} = 0$ and $n_{22} = 4$.

19.2.6.3 Approximation by Permutation Sampling

As the transmission table size and number of marker alleles increases, program execution time of the exact permutation test becomes prohibitively slow. For transmission tables with greater than about 300 observations, or with more than about 8 alleles, the sampling approximation is recommended. Instead of considering every possible permutation, a random sample from the set of all possible permutations, conditional on the observed transmission table, is taken.

The proportion of permutations with significance equal to or greater than the observed table is computed. This proportion is an estimate of the exact pvalue of the observed table. The standard error of the estimated p-value is obtained by computing the variance among several batches of permutations. The total number of permutations considered is chosen to estimate the resulting pvalue within 20% of its true value with 95% confidence.

19.3 Program Input

19.3.1 Running tdtex

A typical run of the TDTEX program may use flags to identify the file types like the following:

>tdtex -p data.par -d data.ped

or, rely on a set file order like the following:

>tdtex data.par data.ped

where data.par is the name of the parameter file and data.ped is the name of the pedigree data file.

19.3.2 The tdtex Block

A tdtex block in the parameter file sets the options on how to perform an analysis using TDTEX. The following table shows the syntax for a tdtex parameter which starts the tdtex block.

The following table lists the parameters and attributes that may occur in a tdtex block.

- 1. The user may list as many different marker parameters as desired. If no marker parameters are specified, then the default TDTEX behavior is to score transmissions for all markers found in the pedigree data file.
- 2. A classic TDT can be performed by setting max_children to 1 and max_sib_pairs to none. All affected children in a pedigree can be used as if they are independent by setting max_children to unlimited and max_sib_pairs to none.
- 3. Regardless of the values of max_children and max_sib_pairs, pedigrees must have at least one typed parent.
- 4. A sibling TDT using only one sib-pair per pedigree can be performed by setting max_children to none and max_sib_pairs to 1. A sibling TDT using all sibling-pairs in a pedigree as if they are independent can be performed by setting max_children to none and max_sib_pairs to unlimited.
- 5. If false, sex of parent is ignored.
- 6. If set to false, the computation time may be excessive. See 19.2.6.3.

19.4 Program Output

TDTEX produces several output files that contain results and diagnostic information:

19.4.1 TDTEX Analysis Output File

One analysis output file, named "tdtex.out", is generated per run of TDTEX. It contains the results of all tests.

Example:

Chapter 20

DESPAIR

DESPAIR is a program to help in designing linkage studies for searching the whole autosomal genome. Originally created for a study comprising affected pairs of relatives of a particular type, the latest version of DESPAIR has been modified to further incorporate discordant relative pairs into the study. The program can be used to determine, for specified power and significance level, the optimal two-stage study design $-$ i.e., how many pairs of relatives should be studied, how many equally spaced markers should be used initially, and what criterion should be used to specify the markers around which further searching should be done. Alternatively, the program will calculate either the number of relative pairs required for a given number of first-stage markers, or the number of markers required for a given number of relative pairs. A novel use of the program DESPAIR has been presented by Ochs-Balcom et al (2010)

Note: The DESPAIR program can only be run on the S.A.G.E. web site at:

http://darwin.cwru.edu/despair/.

20.1 Limitations

The method used assumes that independent pairs of relatives of a single particular type (full sibling, half-sibling, grandparent-grandchild, avuncular, or first cousin) are being sampled. Only three levels of interference are considered, corresponding to Haldane's mapping function (no interference), Kosambi's mapping function (moderate interference), and Morgan's linear mapping function (extreme interference). The spacing between markers is not allowed to be less than one tenth of a centimorgan, nor as much as one morgan, and markers are assumed to be in linkage equilibrium. Two test statistics are allowed for in the cases of sibling pairs, but only one (that based on the mean test) is implemented for designs that use both affected and discordant pairs.

20.2 Theory

It is well understood that linkage of a putative disease locus to a polymorphic marker can be conducted through a study design of affected pairs of relatives, and this is usually the most powerful sampling strategy for binary traits (Blackwelder and Elston, 1985; Risch, 1990). However, recent research shows that, under certain situations, using discordant relative pairs can be as powerful as, or even more powerful than, using affected relative pairs. Moreover, combining discordant with affected relative pairs provides a more valid and reasonable study from both a theoretical and practical point of view (Guo and Elston, 2000). Specifically, linkage can be studied by typing pairs of relatives and examining the proportions of the pairs sharing 0, 1, or 2 alleles identical by descent (IBD) at the marker locus. The test for linkage in DESPAIR is based on either the proportion of pairs sharing 0 alleles IBD or the mean proportion of marker alleles shared IBD, which depend on the type of relative pair.

Denote the expected values of either of these proportions under the null hypothesis of free recombination π_0 . If there is linkage, the expected values are $\pi_0 + \delta_c$ and $\pi_0 - \delta_d$, corresponding to a design using affected relative pairs alone and a design using discordant relative pairs alone, respectively; δ_c and δ_d are the expected deviations respectively for affected pairs and discordant pairs due to linkage. Both these measures depend not only on the type of relative pair, but also on the recombination fraction θ between the marker and disease loci. In addition, δ_c depends on the relative recurrence risk of disease, due to the disease locus, to first degree relatives of affected persons:

> $\lambda = \frac{Pr(\text{first degree relative of affected person is affected})}{Pr(\text{random member of population is affected})}$ *Pr*(random member of population is affected)

and δ_d depends on the corresponding relative non-recurrence risk ratio for an affected-unaffected first degree relative pair:

$$
\lambda^{-} = \frac{Pr(\text{first degree relative of affected person is unaffected})}{Pr(\text{random member of population is unaffected})}.
$$

Each of these relative risks, often called risk ratios, can be to either a parent/offspring (λ_o , λ_o^-) or to a full sibling $(\lambda_s, \lambda_s^{-})$.

If several disease loci act multiplicatively, the relative risk is the product of λ 's, one for each locus. For a study design that combines affected relative pairs with discordant relative pairs, the test statistic is based on the notion that, in the presence of linkage, affected relative pairs are expected to share a larger proportion of marker alleles IBD, whereas discordant relative pairs are expected to share a smaller proportion of alleles IBD. The difference in the proportion of alleles shared IBD between affected pairs and discordant pairs is quantified by Δ , a weighted difference in the deviations of the mean proportions from π_0 . Δ equals zero under the null hypothesis of no linkage, and is greater than zero when linkage is present. The values of Δ can be expressed as a function of θ , λ , λ^- , and the ratio (r_p) of the number of affected relative pairs to the number of discordant relative pairs that are sampled. Values of $\pi_0 + \delta_c$ were given by Risch (1990), and values of $\pi_0 - \delta_d$, and Δ were given by Guo and Elston (2000), for five types of relative pairs: full sibling, half sibling, avuncular, grandparent-grandchild, and first cousin.

The test based on the proportion sharing 0 alleles IBD and the mean test give identical results except in the case of full sib pairs. The test based on the proportion sharing 0 alleles IBD is not implemented for designs using both concordant affected and discordant full sib pairs.

Assume that at a first stage, *m* fully informative markers, equally spaced along an autosomal genome *M* morgans long, are determined on *n* pairs of relatives of a particular type. For each marker, a one-sided test is performed at the α^* significance level to decide whether the sample proportion of alleles shared IBD deviates significantly from π_0 , suggesting linkage. Around each marker suggesting linkage at the first stage, a further 2*k* fully informative markers are tested for linkage at a second stage, assuming that these are placed *(k* on either side of the first stage marker)

.

Figure 20.1: Stage-1 and Stage-2 Marker Placement

to span in an optimal manner the interval of interest suggested by the significant first-stage marker (see Figur[e20.1\)](#page-378-0).

Assume that we want to design a study to have power 1 - β of detecting a disease locus with relative risk ratio λ at a significance level α at the second stage, and that there are actually *d* such disease loci present. Finally, assume that the cost of recruiting a person into the study is *R* times the cost of determining one marker on one person. Under these assumptions, if at most one first stage marker is linked to any disease locus, the expected cost of the study is proportional to

$$
2n\{R+m+2k[\alpha^*m+(1-\beta)d]\}.
$$
 (20.1)

However, because there may be more than one first-stage marker linked to the disease locus, the total expected cost is more appropriately reflected by

$$
C = 2n\{R + m + 2k[\alpha^*(m - \sum_{i=1}^d l_i) + \sum_{i=1}^d \sum_{j=1}^{l_i} (1 - \beta_{ij})]\},
$$
\n(20.2)

where l_i is the number of first stage markers linked to disease locus *i*, and $1 - \beta_{ij}$ is the probability that 2*k* second stage markers are typed around marker *j* that is linked to disease locus *i* (Ziegler et al. 2001). In this revised version of DESPAIR, which implements cost function [\(20.2\)](#page-378-1), users have the option to input a maximum distance (g) between any disease locus and a "linked" marker. Then significant results obtained within *g* morgans from any disease locus are considered to be successes, and any outside that range are considered to be false positives. By making the distance *g* small in comparison to the distance between first stage markers, for a large number of markers cost function [\(20.2\)](#page-378-1) approaches cost function [\(20.1\)](#page-378-2), which was the function used in the original version of DESPAIR.

Suppose the following are specified: α , β , λ , R , d , g , M , the type of relative pairs, and the type of data (affected relative pairs, discordant relative pairs, or both discordant and affected pairs: for the latter two cases, λ^- must also be specified; and for the last one case, the ratio r_p of the number of affected to the number of discordant relative pairs to be sampled must also be specified). Given all this, DESPAIR finds the values of m , n , and α^* that minimize this expected cost for different mapping functions (linear, Kosambi's, and Haldane's), and for values of *k* from 0 (a one-stage design) to a specified maximum value of k , subject to the limitation $M < m < 1000M$ (i.e., the markers must be spaced less than one morgan apart, and must be no closer than one tenth of a centimorgan apart). There is an option to include the cost of screening the population to find the desired sample (the cost of screening is taken to be the same as the cost of recruiting), in which case the user must also enter the proportion of the screened population (r_s) that becomes the final sample.

It is assumed that *n* is large enough, in determining the test criterion corresponding to α^* and β , that the distribution of the proportion of pairs sharing 0 alleles IBD or the mean proportion of marker alleles shared IBD is normally distributed. However, in the case of α , which is typically much closer to zero, there is the option of using either this same approximation assumption (the approximate method), or exact binomial distribution probabilities (the exact method, not implemented for the case where the sample includes both affected and discordant pairs).

To allow for less than fully informative markers, a value of the polymorphism information content (PIC), which measures the markers informativeness (assumed to be the same for all markers), can be specified. This is converted by the program to the corresponding type-of-pair-specific LIC value (Guo and Elston, 1999; Guo et al. 2002). Similarly, a fraction *h* , heterogeneity, can be specified that represents the proportion of the sample pairs affected due to causes other than segregation at the linked locus (in this case one would typically specify a large value for λ and/or a small value of λ^-).

Further details of the method are given in the references.

20.3 Running the Program

DESPAIR can be run by clicking on the DESPAIR GUI link on the S.A.G.E. website (found in the drop-down menu under Programs & Links)

http://darwin.cwru.edu/despair/

and inputting one or more sets of parameters for which the sample size (numbers of relative pairs and/or number of markers) is desired. The parameters may be specified as follows:

Notes

- 1. The method parameter is not applicable for sample data comprising both affected pairs and discordant pairs; only the approximate method (A) is implemented for such data.
- 2. The default value for α corresponds to a lod score of 3 if the method parameter is set to A (approximate).
- 3. The parameter offspr_recurrence_risk and offspr_nonrecurrence_risk are used by the proportion test for linkage, while the parameters sib_recurrence_risk and sib_nonrecurrence_risk are used by the mean test.
- 4. When the value of the screening_cost parameter is set to N, the screened_proportion parameter will be ignored.
- 5. When the value of the screening_cost parameter is set to N, or the concordance_type parameter is set to either A or D, the conc_disc_ratio parameter will be ignored. In other words, the conc_disc_ratio parameter is applicable only when the concordance_type parameter is set to B.
- 6. The user may specify a value for either num_stage_one_markers or num_pairs, but not both. If a value for either one of the parameters is specified, the other will be determined by the program. If neither parameter is specified, the program will determine both.

20.4 Output

DESPAIR produces a Standard Output File that includes:

- Title, version, and date of the program for each problem
- Control values specified by user
- For each $k = 0, \ldots$, max k, and for each mapping function, tabulation of optimal values of m and n with corresponding α^* , cost (in units of the cost of typing one marker on one person), and the first and second stage marker spacings in centimorgans

20.4.1 Error Messages

DESPAIR has an error checking routine. Values of any parameter that are out of bounds are not allowed. When an error is detected during the analysis, DESPAIR will identify the error and display the error message associated with it. The error messages that may be displayed are as follows:

- The following fields were set to values out of bounds: <FIELD LIST>
- The exact test is not implemented for the case in which both concordant and discordant pairs are available.
- The test based on the proportion sharing 0 alleles i.b.d. is not available. The above results are for the mean test.

Chapter 21

References

Amos CI, Dawson DV, Elston RC. (1990) *The Probabilistic Determination of Identity-by-Descent Sharing for Pairs of Relatives from Pedigrees.* American Journal of Human Genetics; 47:842-853

Bickeboller H, Clerget-Darpoux F. (1995) *Statistical Properties of the Allelic and Genotypic Transmission/Disequilibrium Test for Multiallelic Markers*. Genetic Epidemiology; 12(6):865-870

Blackwelder WC, Elston RC. (1985) *A comparison of sib-pair linkage tests for disease susceptibility loci*. Genetic Epidemiology; 2:85-97

Boehnke M. (1991) *Allele Frequency Estimation from Data on Relatives*. American Journal of Human Genetics 48:22-25

Bonney GE. (1984) *On the statistical determination of major gene mechanisms in continuous human traits: regressive models*. American Journal of Medical Genetics; 18:731-749

Bonney GE. (1986) *Regressive logistic models for familial disease and other binary traits*. Biometrics; 42:611-625

Bonney GE. (1998) *Regressive Models*. Encyclopedia of Biostatistics; Vol 5:3755-3762

Box GEP, Cox DR. (1964) *An analysis of transformations*. Journal of the Royal Statistical Society [B]; 26:211-252

Cannings C, Thompson EA, Skolnick MH. (1978) *Probability functions on complex pedigrees*. Advanced Applied Probability; 10:26-61

Carroll RJ, Ruppert D. (1984) *Power Transformations When Fitting Theoretical Models to Data*. American Journal of the Statistical Association; 79:321-328

Cleves MA, Olson JM, Jacobs KB. (1997) *Exact Transmission-Disequilibrium Tests with Multiallelic Markers*. Case Western Reserve University School of Medicine Internal Paper

Chen H, Chen J, Kalbfleisch JD (2001) *A Modified Likelihood Ratio Test for Uncertain-Haplotype Transmission*. Journal of the Royal Statistical Society (B); 63:19-29

Curtis D and Sham PC. (1995) *An Extended Transmission/Disequilibrium Test (TDT) for Multi-Allele Marker Loci.* Annals of Human Genetics; 59:323-336

Demenais FM, Murigande C, Bonney GE. (1990) *Search for faster methods of fitting the regressive models to quantitative traits*. Genetic Epidemiology; 7:319-334

Demenais FM, Elston RC. (1981) *A General Transmission Probability Model for Pedigree Data.* American Journal of Human Genetics; 33:300-306

Elston RC. (1992) *Designs for the global search of the human genome by linkage analysis.* In: Proceedings of the XVIth International Biometric Conference, Hamilton, New Zealand, December 7-11, 1992, pp 39-51.

Elston RC, Guo X, Williams L. (1996) *Two-stage global search designs for linkage analysis using pairs of affected relatives.* Genetic Epidemiology; 13:535-558.

Elston RC, Stewart J. (1971) *A general model for the genetic analysis of pedigree data*. Human Heredity; 21:523-542

Elston RC, Bonney GE. (1986) *Sampling via Probands in the Analysis of Family Studies*. Proceedings of the 13*th* International Biometric Conference

Elston RC, George VT, Severtson F. (1992) *The Elston-Stewart algorithm for continuous genotypes and environmental factors*. Human Heredity; 42:16-27

Feingold E, Brown PO, Siegmund S. (1993) *Gaussian Models for Genetic Linkage Analysis Using Complete High-Resolution Maps of Identity by Descent.* American Journal of Human Genetics; 53:234-251

Fernando RL, Stricker C, Elston RC. (1993) *An Efficient Algorithm to Compute Posterior Genotypic Distribution for Every Member of a Pedigree Without Loops.* Theory of Applied Genetics; 87:89-93

Fernando RL, Stricker C, Elston RC. (1994) *The finite polygeinc mixed model: An alternative formulation for the mixed model of inheritance*. Theort of Applied Genetics; 88:573-580

George VT, Elston RC. (1987) *Testing the association between polymorphic markers and quantitative traits in pedigrees*. Genetic Epidemiology; 4:193-201

George VT, Elston RC (1988) *Generalized modulus power transformations*. Commun Statististics – Theory Methodology; 17:2933-2952

George VT et. al. (1999) *A Test of Transmission/Disequilibrium for Quantitative Traits in Pedigree Data, by Multiple Regression*. American Journal of Human Genetics; 65:236-245

Ginsburg E, Malkin I, Elston RC. (2006) *Theoretical Aspects of Pedigree Analysis.* Tel-Aviv, Israel. Ramot Publishing House.

Go RCP, Elston RC, Kaplan EB. (1978) *Efficiency, robustness of pedigree segregation analysis*. American Journal of Human Genetics; 30:28-37

Goddard KAB, Witte JS, Suarez, BK, Catalona, WJ, Olson, JM. (2001) *Model-free Linkage Analysis with Covariates Confirms Linkage of Prostate Cancer to Chromosomes 1 and 4*. American Journal of Human Genetics; 68:1197-1206

Guo X, Elston RC. (1999) *Linkage information content of polymorphic genetic markers.* Human Heredity; 49:112-118

Guo X, Elston RC. (2000) *Two-stage global search designs for linkage analysis II: Including discordant relative pairs in the study.* Genetic Epidemiology; 18:111-127

Guo X, Olson JM, Elston RC, Niu T. (2002) *The linkage information content value of polymorphism genetic markers in model-free linkage analysis.* Human Heredity; 53:45-48.

Hanson R, Knowler W. (1998) *Analytic Strategies to detect linkage to a common disorder with genetically determined age of onset.* Genetic Epidemiology; 15:299-315

Idury RM, Elston RC. (1996) *A Faster and More General Hidden Markov Model Algorithm for Multipoint Likelihood Calculations.* Human Heredity; 47: 197-202

Ito et al. (2003) *Estimation of Haplotype Frequencies, Linkage-Disequilibrium Measures, and Combination of Haplotype Copies in Each Pool by Use of Pooled DNA Data*. American Journal of Human Genetics; 72(2):384-398

Karunaratne PM, Elston RC. (1998) *A multivariate logistic model (MLM) for analyzing binary family data*. American Journal of Medical Genetics; 76:428-437

Kuglyak L, Lander ES. (1995) *Complete Multipoint Sib-Pair Analysis of Qualitative and Quantitative Traits.* American Journal of Human Genetics; 57: 439-454

Lander ES, Green P. (1987) *Construction of Multilocus Genetic Maps in Humans.* Proceedings National Academy of Science USA; 84:2363-2367

Lange K. (1997) *An Approximate Model of Polygenic Inheritance*. Genetics; 147:1423-1430

Lange K, Elston RC. (1975) *Extensions to Pedigree Analysis I-Likelihood Calculations for Simple and Complex Pedigrees*. Human Heredity; 25:95-105

Mathew G, Song Y, Elston RC. (2011) *Interval estimation of familial correlations from pedigrees*. Statistical Applications in Genetics and Molecular Biology; 10(1): Article 11

McCulloch CE, Neuhaus, JM. (2011) Misspecifying the Shape of a Random Effects Distribution: Why Getting It Wrong May Not Matter. Statistical Science; 26(3): 388–402

Ochs-Balcom HM, Guo X, Yonebayashi T, Wiesner G, Elston RC. (2010) *Program Update and Novel Use of the DESPAIR Program to Design a Genome-Wide Linkage Study Using Relative Pairs*. Human Heredity; 69(1): 45-51

Olson JM, Wijsman EM. (1993) *Linkage between quantitative trait and marker loci: methods using all relative pairs*. Genetic Epidemiolgy; 10:87-102

Olson JM, Jacobs KB, Cleves MA. (1997) *Exact tests of table symmetry*. Internal paper

Olson JM. (1999) *A General Conditional-Logistic Model for Affected-Relative-Pair Linkage Studies*. American Journal of Human Genetics; 65:1760-1769

Olson JM, Song Y, Lu Q, Wedig GC, Goddard KB. (2004) *Using overall allele-sharing to detect the presence of large-scale data errors and parameter misspecification in sib-pair linkage studies*. Human Heredity; 58:49-54

Parzen E. (1962) *On Estimation of a Probability Density Function and Mode.* Annals of Mathematical Statistics; 33:1065-1076

Pericak-Vance MA, Elston RC, Conneally PM, Dawson DV. (1983) *Age-of-Onset Heterogeneity in Huntington's Disease Families*. Journal of Human Genetics; 14:49-59

Quade SR, Elston RC, Goddard KA. (2005) *Estimating Haplotype Frequencies in Pooled DNA Samples when there is Genotyping Error*. BMC Genetics; 6(1):25

Rice JP, Neuman RJ, Hoshaw SL, Daw EW, Gu C. (1995) *TDT with covariates and genomic screens with mod scores: their behavior on simulated data*. Genet Epidemiol; 12:659-664

Risch, N. (1987) *Assessing the role of HLA-linked and unlinked determinants of disease.* American Journal of Human Genetics; 40:1-14

Risch N. (1990) *Linkage strategies for genetically complex traits. II. The power of affected relative pairs.* American Journal of Human Genetics; 46:229-241.

Risch, N. (1990) *Linkage Strategies for Genetically Complex Traits. III. The effect of Marker Polymorphism on Analyses of Affected Relative Pairs.* American Journal of Human Genetics; 46:242- 253

Schnell AH, Sun X, Igo RP, Elston RC. (2012) *Some capabilities for model-based and model-free linkage analysis using the program package S.A.G.E. (Statistical Analysis for Genetic Epidemiology)*. Human Hereidty; in press

Scott D, Szewczyk W. (2000) *Fitting Mixtures of Regression Models by L2E*

Self S, Liang K. (1987) *Asymptotic properties of maximum likelihood estimators and likehood ratio tests under nonstandard conditions*. Journal of the American Statatistica Association; 82:605-610

Sinha M, Song Y, Elston RC, Olson JM, Goddard KAB. (2006) *Prediction of empirical p values from asymptotic p values for conditional logistic affected relative pair linkage analysis*. Human Hereidty; 61(1): 45-54

Sobel E, Lange K. (1996) *Descent Graphs in Pedigree Analysis: Applications to Haplotyping, Location Scores, and Marker-Sharing Statistics.* American Journal of Human Genetics; 58:1323-1337

Spielman RS, Ewens WJ. (1996) *The TDT and Other Family-Based Tests for Linkage Disequilibrium and Association*. American Journal of Human Genetics; 59:983-989

Spielman RS, McGinnis RE, Ewens WJ. (1993) *Transmission Test for Linkage Disequilibrium: The Insulin Gene Region and Insulin-Dependent Diabetes Mellitus (IDDM).* American Journal of Human Genetics; 52:506-516

Wang N, Akey JM, Zhang K, Chakraboryy R, Jin L. (2002) *Distribution of Recombination Crossovers and the Origin of Haplotype Blocks: The Interplay of Population History, Recombination and Mutation*. American Journal of Human Genetics; 71:1227-1234

Wang S, Kidd KK, Zhao H. (2003) *On the Use of DNA Pooling to Estimate Haplotype Frequencies*. Genetic Epidemiology; 24:74-82

Wang T, Elston RC. (2005) *Two-level Haseman-Elston regression for general pedigree data analysis*. Genetic Epidemiology; 29:12-22

Wang T, Elston RC. (2006) *A quantitative linkage score for an association study following a linkage analysis*. BMC Genetics; 7:5

Wang T, Elston RC. (2007) *Regression-based multivariate linkage analysis with an application to blood pressure and body mass index*. Annals of Human Genetics; 71:96-106

Wijsman EM, Amos CI. (1997) *Genetic Analysis of Simulated Oligogenic Traits in Nuclear and Extended Pedigrees: Summary of GAW10 Contributions*. In: Goldin L, Bailey-Wilson J, Borecki I, Falk C, Goldstein A, Suarez B, and MacCluer J. *Genetic Analysis Workshop 10: Detection of genes for complex traits*. Genetic Epidemiology*;* 14:S719-S736

Whittemore AS, Tu IP. (1998) *Simple, robust linkage tests for affected sibs*. American Journal of Human Genetics; 62:1228-1242

Ziegler A, Böddeker I, Geller F, Müller H, Guo X. (2001) *On the total expected study cost in twostage genome-wide search designs for linkage analysis using the mean test for affected sib pairs.* Genetic Epidemiology; 20:397-400.

Zhu X, Olson JM, Schnell AH, Elston RC. (1997) *Genetic Analysis Workshop 10: Model-free ageof-onset methods applied to the linkage of bipolar disorder.* Genetic Epidemiology; 14:711-716.